Barcodes and QR-Codes

Examples of error detection and correction

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Abstract

This work focuses on the creation of barcodes and QR-Codes. It describes and explains the different data encodings and algorithms which make such technologies possible. Following the introduction, the second part is about Code-39 and EAN barcodes, and the third about QR-Codes. Then, the fourth chapter presents in more details some methods for error detection and correction. The final section introduces a new custom type of code named Lycacode which relies upon some aspects seen in the previous three chapters. Additionally, many parts are implemented in Python like the generation of QR-Codes and barcodes for example. These can either be found in the appendices or in the associated GitHub repository[1].

[1] https://github.com/LordBaryhobal/5D_Heredero_Louis_TM2022/tree/main/python

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Chapter 1

Introduction

Computers and microprocessors are certainly the defining innovations of the end of the XXth and beginning of the XXIst centuries. Invented to perform tasks faster and more reliably than humans, they truly have surpassed our mental capacities in many areas. Although they overcome a great number of our shortcomings, especially in terms of speed, they are not infallible. While many computer related bugs can be tracked down to a human error, some are inherent to the physical infrastructure of our technologies. One major material limitation is the network connecting computers to each other. For example, WiFi and mobile data use radio transmissions which are not perfectly reliable. Similarly, space communication implies a lot of interferences due to the atmosphere, space debris and all sorts of radiations. Invalid data may also come from external sensors, would that be because they are malfunctioning or simply not able to correctly interpret there inputs.

As such, engineers and programmers have to devise methods to check that the data received is unaltered and provide a way of recovering the original information, or at least guess it. These methods are most useful in fields where data is, or was, manually input. As the saying goes, to err is human, and machines help us correct these errors. Of course, these concerns are not new. Indeed, Claude Shannon had already started taking an interest in information theory in 1948[6]. This scientific field which studies the characteristics and behaviors of information led to many technological improvements and fundamental theories, especially in computer science.

Barcodes and QR-Codes are two instances of the consequences and use of information theory. Their main benefit is to allow reliable identification of objects by computers. Prior to these inventions, it was the task of humans to manually tell machines what an object was, for example in supermarkets or factories. Now, a simple camera can quickly recognize items, without the use of artificial intelligence, which requires substantially higher computing capacity. Additionally, their simple designs make them very easy and efficient to implement, even on limited hardware.

Chapter 2

Barcodes

2.1 Origin

The patent for the first barcode[13] was filed slightly more than 70 years ago, in 1949, by then Drexel University students Norman J. Woodland and Bernard Silver. Officially established in 1952, this patent described the first barcode, a reading device and a circular design. However, success did not strike immediately, mainly because of the impractical and limited resources of the time. Its first use was on trains, as an identification system (KarTrak[10]). It was soon abandonned however because of readability and maintenance problems.

A few years later, in the 1970s, IBM, which now counted Mr. Woodland as an associate, had developed the linear UPC barcode. The Universal Product Code is still in use today, with some modifications and standardizations. It was and is used in stores to identify groceries. This kind of tagging allowed for shorter waits at registers when checking out and greatly influenced the capitalist society we live in nowadays. As well as improving time efficiency, it also reduced the number of human errors which could happen when manually entering the prices of items bought by customers.

Since then, many other types of barcodes have been created for more specialized purposes, like postal mail, warehouse inventories, libraries or medicines. The main advantage of barcodes is that they can be read very quickly with a single laser scan. The first barcodes didn't even need a computer and could be decoded with only relatively simple electronic circuits [13, p.3].

In 1974, the GS1 was founded. This international group is responsible for managing encoding standards in the field of logistics and sale of goods.

2.2 How it works

In barcodes, black and white stripes encode data visually. For certain types of codes, they represent 1s and 0s, while for other types such as Code-39, it is the width of the stripes which determines the value. The following sections will explain more thoroughly some of the most used types of barcodes.

2.2.1 Code-39

First of all, there is Code-39, one of the simplest type of barcode. It can encode 43 different alphanumerical characters plus the special '*' delimiter. Each character is assigned a particular group of 9 bits, 3 of which are 1s, hence the name. Table 2.1 lists all encodable characters and their respective codes.

Char	Code	Char	Code	Char	Code
A	100001001	P	001010010	4	000110001
B	001001001	Q	000000111	5	100110000
C	101001000	R	100000110	6	001110000
D	000011001	S	001000110	7	000100101
E	100011000	T	000010110	8	100100100
F	001011000	U	110000001	9	001100100
G	000001101	V	011000001	*space*	011000100
H	100001100	W	111000000	-	010000101
I	001001100	X	010010001	$	010101000
J	000011100	Y	110010000	%	000101010
K	100000011	Z	011010000	.	110000100
L	001000011	0	000110100	/	010100010
M	101000010	1	100100001	+	010001010
N	000010011	2	001100001	*	010010100
O	100010010	3	101100000		

Table 2.1: Code-39 characters

To create a Code-39 barcode, we just have to take the codes corresponding to each character and join them end to end, adding a 0 in between each group. Finally, we add the '*' character at both ends (also spaced by a 0). It is a delimiter and should always be present at the beginning and end of the code, to signal scanners that it is a Code-39 barcode, as well as providing a reference for the normal bar width.

Ones represent wide bars and zeros thin bars[1]. Black and white stripes alternate, starting with black.

Let's encode the string "CODE-39"

Char	Code
*	010010100
C	101001000
O	100010010
D	000011001
E	100011000
-	010000101
3	101100000
9	001100100
*	010010100

Table 2.2: Code-39 for "*CODE-39*"

For example, the first delimiter is encoded as:

The entire barcode would look like figure 2.1

Figure 2.1: Code-39 example ("*CODE-39*")

[1] The ratio wide/thin must be comprised between 2:1 and 3:1 [5]

Code-39 doesn't provide any error detection mechanism. If a character is read as another character, the reading device won't know that it made a mistake. Additionally, it is not the most compact type of encoding, but it can easily be used for small amounts of data, for example to encode the identification number on students' absence booklet or sheet.

2.2.2 EAN

Another barcode type is the EAN[2] standard. It is used for product identification and can be found on anything bought from a store. It exists in two main formats: EAN-8 and EAN-13, encoding respectively 7 and 12 digits. EAN codes use what is called a check digit to detect erroneous readings. This digit is the result of the Luhn Formula. Say we want to encode the number `978294062105`. To find the checksum digit, we have to multiply each digit by the alternating factors 1 and 3, and add all the products.

	9	7	8	2	9	4	0	6	2	1	0	5	
x	1	3	1	3	1	3	1	3	1	3	1	3	
+	9	21	8	6	9	12	0	18	2	3	0	15	= 103

Table 2.3: Luhn Formula

We then take the modulo ten, which is the same as saying we keep the unit's digit, and subtract it from 10. In our example, $103 \equiv 3 \pmod{10}$ so the checksum is $10 - 3 = 7$. If we get 10 we change it to 0 to keep a single digit.

This is now the 13th digit of our code. For an EAN-8 code, the process is the same with the factors 1 and 3 inverted, meaning the first digit is multiplied by 3, the second by 1, etc.

The barcode itself is built out of a series of "elements". Each element encodes a character, as described by table 2.4. Ones are black stripes and zeros are white.

There are 3 types of elements: A, B and C. Type C is obtained by swapping 1s and 0s from type A. Type B is obtained by flipping type C from left to right.

This structure makes it easy to find the type of a certain element. Bs and Cs have an even number of 1s, while As have an odd number. Additionally, As an Bs start with 0 and end with 1, whereas Cs are the opposite.

In this way, if the code is read in the wrong way, C elements will appear as Bs, and A elements will be invalid because no element starts with a 1 and has an odd number of 1s. Similarly, if the barcode is printed with inverted colors (white on black), A elements will be read as Cs, and B elements will be invalid, since no element starts with a 1 and has an odd number of 1s.

EAN barcodes are thus very practical since they can be scanned in any orientation and support color inversion.

[2] European Article Numbering

Char.	A	B	C
0	0001101	0100111	1110010
1	0011001	0110011	1100110
2	0010011	0110011	1101100
3	0111101	0110011	1000010
4	0100011	0110011	1011100
5	0110001	0110011	1001110
6	0101111	0110011	1010000
7	0111011	0110011	1000100
8	0110111	0110011	1001000
9	0001011	0110011	1110100

Table 2.4: EAN elements

EAN-8

An EAN-8 barcode has the following structure:

left	A	A	A	A	mid	C	C	C	C	right

Table 2.5: EAN-8 structure

In table 2.5, ”left” and ”right” are the end delimiters ”101”. ”mid” is the center delimiter ”01010”. To illustrate, let’s encode the value ”8427372”.

1. Calculate the checksum digit:

	8	4	2	7	3	7	2	
x	3	1	3	1	3	1	3	
+	24	4	6	7	9	7	6	= 63

Table 2.6: Luhn Formula (EAN-8 example)

   Thus the last digit is $10 - (63 \bmod 10) = 10 - 3 = 7$.

2. Take each digit’s corresponding element: As for the first 4, Cs for the rest (table 2.7).
3. Add the delimiters ”101” at each end and ”01010” between both halves of the code.

4. For each 1, draw a black bar and for each 0 a white one (figure 2.2).

Char.	Element
8	0110111
4	0100011
2	0010011
7	0111011
3	1000010
7	1000100
2	1101100
7	1000100

Table 2.7: EAN-8 example elements

Figure 2.2: EAN-8 example ("84273727")

EAN-13

EAN-13 follows the same principles as EAN-8. The structure of such a code is the following:

left	A	A	A	A	A	A	mid	C	C	C	C	C	C	right
	B	B	B	B	B	B							

Table 2.8: EAN-13 structure

This has only 12 places for elements, 6 A/B and 6 C. The 13th digit (in reality the first) is encoded in the pattern of A and B elements. Here is the list of patterns corresponding to each digit:

Digit	Pattern	Digit	Pattern
0	AAAAAA	5	ABBAAB
1	AABABB	6	ABBBAA
2	AABBAB	7	ABABAB
3	AABBBA	8	ABABBA
4	ABAABB	9	ABBABA

Table 2.9: EAN-13 1st digit patterns

It can be noticed that the first element is always an A so that reading direction can easily be determined.

Let's illustrate EAN-13 by encoding the value "978294062105".

1. Calculate the checksum digit:

	9	7	8	2	9	4	0	6	2	1	0	5	
x	1	3	1	3	1	3	1	3	1	3	1	3	
+	9	21	8	6	9	12	0	18	2	3	0	15	= 103

Table 2.10: Luhn Formula (EAN-13 example)

   Thus the last digit is $10 - (103 \text{ mod } 10) = 10 - 3 = 7$.

2. Get the pattern corresponding to the first digit: 9 $\rightarrow$ ABBABA
3. Take the corresponding element for each digit

Char.	Type	Element	Char.	Type	Element
7	A	0111011	6	C	1010000
8	B	0001001	2	C	1101100
2	B	0011011	1	C	1100110
9	A	0001011	0	C	1110010
4	B	0011101	5	C	1001110
0	A	0001101	7	C	1000100

Table 2.11: EAN-13 example elements

4. Add the delimiters "101" at each end and "01010" between both halves of the code.
5. For each 1, draw a black bar and for each 0 a white one.

Figure 2.3: EAN-13 example ("9782940621057")

2.3 Application in Python

In this section, we will implement barcode generation in Python. We will first program a ”Code-39” encoder, then an EAN-8 and finally an EAN-13.

2.3.1 Code-39

This type of code being just a matter of translating each character to a particular group of wide and narrow stripes, the implementation is quite simple.

We first create a dictionary holding the codes for each character.

```
code39_dict = {
    "A": "100001001", "B": "001001001",
    "C": "101001000", "D": "000011001",
    "E": "100011000", "F": "001011000",
    "G": "000001101", "H": "100001100",
    "I": "001001100", "J": "000011100",
    "K": "100000011", "L": "001000011",
    "M": "101000010", "N": "000010011",
    "O": "100010010", "P": "001010010",
    "Q": "000000111", "R": "100000110",
    "S": "001000110", "T": "000010110",
    "U": "110000001", "V": "011000001",
    "W": "111000000", "X": "010010001",
    "Y": "110010000", "Z": "011010000",
    "0": "000110100", "1": "100100001",
    "2": "001100001", "3": "101100000",
    "4": "000110001", "5": "100110000",
    "6": "001110000", "7": "000100101",
    "8": "100100100", "9": "001100100",
    " ": "011000100", "-": "010000101",
    "$": "010101000", "%": "000101010",
    ".": "110000100", "/": "010100010",
    "+": "010001010", "*": "010010100"
}
```

To convert a string, we map each character to its corresponding binary representation and join the resulting codes with ”0” in between.

```
def code39(text):
    text = text.upper()
    text = map(lambda c: code39_dict[c], text)
    return "0".join(text)
```

We will also need a function to render the barcode. For this, we will use the Pygame module.

```
def draw_barcode(barcode, win):
    barcode = list(map(int, barcode))
    width = win.get_width()*0.8
    height = win.get_height()*0.5
    thicks = sum(barcode)
    thins = len(barcode)-thicks
    bar_w = width/(thicks*2+thins)

    win.fill((255,255,255))
    x = win.get_width()*0.1
    y = win.get_height()*0.25

    for i, c in enumerate(barcode):
        w = 2*bar_w if c else bar_w
        if i%2 == 0:
            pygame.draw.rect(win, (0,0,0), [x, y, w, height])

        x += w
```

The full python script can be found in appendix B or on GitHub[3].

2.3.2 EAN-8

The first step to create an EAN-8 barcode is to compute the check digit with Luhn's formula. To make this function also usable for EAN-13, we need to redefine the formula as such:

1. Multiply each digit by the alternating factors 1 and 3 starting with 3 **from the end**.
2. Add them together then take the modulo ten and subtract the result from 10.
3. If the result is equal to 10, change it to 0.

In python, the function multiplies the i^{th} to last digit by:

$$\text{factor} = 3 - (i \; mod \; 2) * 2$$

which basicly is step 1 above.

i	...	4	3	2	1	0
factor	...	3	1	3	1	3

Table 2.12: Python Luhn formula example

[3] https://github.com/LordBaryhobal/5D_Heredero_Louis_TM2022/blob/main/python/code39.py

```
def luhn(digits):
    checksum = sum([
        digits[-i-1]*(3-i%2*2)
        for i in range(len(digits))
    ])
    ctrl_key = 10 - checksum%10
    if ctrl_key == 10:
        ctrl_key = 0

    return ctrl_key
```

Both code types also need the table of elements:

```
A = [
    0b0001101,
    0b0011001,
    0b0010011,
    0b0111101,
    0b0100011,
    0b0110001,
    0b0101111,
    0b0111011,
    0b0110111,
    0b0001011
]

# XOR 0b1111111
C = list(map(lambda a: a^127, A))

# Reverse bit order
B = list(map(lambda c: int(f"{c:07b}"[::-1], 2), C))
```

The following function converts a number to the list of its bits:

```
def bin_list(n):
    return list(map(int, f"{n:07b}"))
```

Finally, the encoding function:

```
def ean8(digits):
    digits.append(luhn(digits))
    elmts = []
```

```
    elmts += [1,0,1] #delimiter
    for digit in digits[:4]:
        elmts += bin_list(A[digit])

    elmts += [0,1,0,1,0] #middle delimiter
    for digit in digits[4:]:
        elmts += bin_list(C[digit])

    elmts += [1,0,1] #delimiter
    return elmts
```

We will use a similar function as in subsection 2.3.1 to render the barcode

```
def draw_barcode(barcode, win):
    width = win.get_width()*0.8
    height = win.get_height()*0.5
    bar_w = width/len(barcode)

    win.fill((255,255,255))
    x = win.get_width()*0.1
    y = win.get_height()*0.25

    for c in barcode:
        if c:
            pygame.draw.rect(win, (0,0,0), [x, y, bar_w, height])

        x += bar_w
```

The full python script can be found in appendix C or on GitHub[4]

2.3.3 EAN-13

The main difference with EAN-8 is the encoding of the first digit, using an A/B pattern. We will create a list of these patterns:

```
ean13_patterns = [
    "AAAAAA",
    "AABABB",
    "AABBAB",
    "AABBBA",
    "ABAABB",
    "ABBAAB",
```

[4] https://github.com/LordBaryhobal/5D_Heredero_Louis_TM2022/blob/main/python/ean.py

```
    "ABBBAA",
    "ABABAB",
    "ABABBA",
    "ABBABA"
]
```

And the appropriate encoding function:

```
def ean13(digits):
    pattern = ean13_patterns[digits[0]]
    digits.append(luhn(digits))
    elmts = []

    elmts += [1,0,1] #delimiter
    for d in range(1,7):
        _ = A if pattern[d-1] == "A" else B
        digit = digits[d]
        elmts += bin_list(_[digit])

    elmts += [0,1,0,1,0] #middle delimiter
    for digit in digits[7:]:
        elmts += bin_list(C[digit])

    elmts += [1,0,1] #delimiter
    return elmts
```

The full python script can be found in appendix C or on GitHub[5]

[5]https://github.com/LordBaryhobal/5D_Heredero_Louis_TM2022/blob/main/python/ean.py

Chapter 3

QR-Codes

3.1 Origin

Eventhough barcodes have conquered the world, they still have some major issues. First of all, their capacity is rather limited, allowing only a maximum of about twenty characters to be encoded in a practical format. Secondly, they can only store a small group of characters, some even only numbers. And finally, they require the reading device to be in a roughly parallel orientation with respect to the code in order to read it.

For these reasons, the Japanese company named DENSO WAVE started developing a new type of 2D code. Indeed, a barcode is one-dimensional - that is, the information is encoded on a single axis. What DENSO WAVE tried was a two-dimensional matrix of data.

The result of their research and development became the well-known ”QR-Code”, which stands for Quick-Response code. They were first used in Toyota factories to track car parts[3]. With the desire to offer this technology to the largest number of people, the company decided not to keep it private but rather make it open-source. It later became a norm in many countries and is now specified by the ISO/IEC 18004 standard [4].

QR-Codes address all the above-mentioned problems related to barcodes.

Data type

They allow the encoding of either numbers, text, raw binary data and even Kanji characters[1]. Therefore they can be used in a great variety of contexts, for example to identify objects, to conviniently share a URL or even a small image.

[1] Kanji are the characters used to write Japanese

Information density

They allow dense information storage.

Type	Number of ”characters”
Numerical	7’089
Alphanumerical	4’296
Bytes	2’953
Kanji	1’817

Table 3.1: Maximum amount of data in a QR-Code [4]

The alphanumerical type can only encode the following characters:

`0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZ $%*+-./:`

Reading freedom

Thanks to finder patterns, they can be read in any orientation, even with perspective. Taking into account the great progress which happened during the last 30 years in the field of cameras and mobile phones, QR-Codes can now be scanned in a matter of milliseconds, no matter the angle of the camera.

Error detection and correction

The other main advantage of QR-Codes is the embedded error detection and correction system.

They come in four different levels of error correction:

- L (low): 7%
- M (medium): 15%
- Q (quartile): 25%
- H (high): 30%

A higher level indicates a greater amount of redundancy and an ability to recover a greater part of a damaged code. This principle is often used to add a custom icon at the center of the QR-Code, since the reader will still be able to scan it and reconstruct the hidden part.

3.2 How it works

Graphically, QR-Codes are composed of black and white squares called "modules". Each module represents a binary bit, black meaning 1, white meaning 0.

Similarly to section 2.2, we will only describe the encoding phase.

To make the following explanations easier to follow, we will use a concrete value.

Let's encode the string "Hello, World!" with level "M" of error correction.

3.2.1 Data type

The first step is to choose the appropriate data type for encoding. In our case, we can't use the numerical format since there are letters, nor can we use the alphanumerical, because of the exclamation mark and lowercase letters. Thus, the most suitable encoding is the byte format.

Note that these different formats exist to optimize encoding to take the least possible space, so it is recommended to choose the minimum required format to avoid unecessarily big QR-Codes.

The first 4 bits of data in our code will be the format used. Numerical is 1, alphanumerical is 2, byte is 4 and kanji is 8. In our case, the format indicator will be "0100".

3.2.2 Version

QR-Codes come in a number of sizes, called "versions". Version 1 (the smallest) is a 21x21 grid, version 2 a 25x25, version 3 29x29, and so on, up to version 40 (the largest) which is a 157x157 matrix.

To know which size our code will be, we have to refer to table D.2 indicating which version is needed for a certain amount of data.

Our string contains 13 characters, that is 13 bytes (in ISO-8859-1) and will be encoded using the byte format with level "M" of error correction. Thus the final code will be of version 1.

3.2.3 Character count indicator

Before encoding our data, we need to create a header stating the total character count. For that, the length is converted to its binary representation of n bits, where n depends on the version and encoding mode, as shown in table 3.2.

Version	Num.	Alpha.	Byte	Kanji
1 to 9	10	9	8	8
10 to 26	12	11	16	10
27 to 40	14	13	16	12

Table 3.2: Bit length of character count indicator

In our example, the number of characters is 13, which means the character count indicator is ”00001101”.

3.2.4 Data encoding

The following step is to convert the data to binary. The method used differs for each format.

Numerical

1. Split the number into 3-digit groups
2. For each group, convert the number to binary, padded to:
    - 10 bits if there are 3 digits (most groups)
    - 7 bits if there are 2 digits (only sometimes for the last group)
    - 4 bits if there is only one digit (only sometimes for the last group)
3. Join the resulting bits end to end

Alphanumerical

1. Split the string into 2-character groups
2. For each group:
    (a) Take the first character’s index in list D.1 and multiply it by 45
    (b) Take the second character’s index in list D.1
    (c) Add them together and convert the result to an 11-bit number
3. If the string has an odd number of characters, take the index of the last character and convert it to a 6-bit number

Byte

1. Encode the data in ISO-8859-1 (latin-1)
2. Join the 8-bit binary representation of each character end to end

Kanji

1. Encode the data in JIS X 0208 (each character is encoded on 2 bytes)
2. For each character (= pair of bytes):
   (a) If the value is between 0x8140 and 0x9FFC, subtract 0x8140
   Otherwise, if the value is between 0xE040 and 0xEBBF, subtract 0xC140
   (b) Multiply the most significant byte by 0xC0
   (c) Add the most significant byte to the least significant
   (d) Join the 13-bit binary representation of each sum end to end

Example

For our example, following the byte encoding format, we get:

01000000	**1101**0100	10000110	01010110	11000110
11000110	11110010	11000010	00000101	01110110
11110111	00100110	11000110	01000010	0001

Note that the format and character count indicator have been added at the begginning (bolded bits).

The resulting binary string needs to be padded before continuing to the next step. This is done in a fourfold process:

1. Get the total number of bits in the final code by multiplying column "Data codewords" of table D.3 by eight[2]. Let that be B and let b be the number of data bits we already have
2. Add $B - b$ 0s, but at most 4
3. Add 0s so that b is a multiple of 8, if not already
4. If $b < B$, fill the remaining bits with the alternating bytes "`11101100`" and "`00010001`"

[2] A codeword is equivalent to an 8-bit byte

In our example, we already have 116 data bits on 128 as per table D.3 (version 1, level M). Thus we add four 0s, increasing b to 120. We don't need to add other 0s since it is already a multiple of 8. Finally we fill the remaining 8 bits with the alternating padding bytes. The result is:

01000000	11010100	10000110	01010110	11000110
11000110	11110010	11000010	00000101	01110110
11110111	00100110	11000110	01000010	00010000
11101100				

3.2.5 Error correction

Now that we have encoded our data, we need to create additional error correction codewords. QR-Codes use what is called the Reed-Solomon algorithm to detect and correct potential errors in a scanned code. This algorithm is explained in more details in section 4.2.

According to table D.3, we need a certain number of error correction codewords (column "Error correction codewords per block"). Let this number be n.

For that, we create a generator polynomial:

$$\prod_{i=0}^{n-1}(x+2^i)$$

Note that the calculation are done in a Galois field, as explained in section 4.2.

Encoded data now needs to be split in B1 blocks (see table D.3, column "Blocks in group 1"). Each block contains C1 data codewords (see table D.3, column "Data codewords per group 1 blocks").

For each block:

1. Convert each codeword to its decimal value (in the Galois field) and let that be the coefficients of a "message" polynomial.
2. Divide this polynomial by the generator polynomial created earlier.
3. Convert the coefficients of the remainder to their 8-bit binary representation and let these be the error correction codewords for this block.

If group 2 has a non-null amount of data codewords, do the same steps for column "Blocks in group 2" (B2) and "Data codewords per group 2 blocks" (C2).

For our example, n = 10, B1 = 1, C1 = 16, B2 = 0, C2 = 0. The generator polynomial has the coefficients:

$$1, 216, 194, 159, 111, 199, 94, 95, 113, 157, 193$$

We only have one block with 16 codewords, so our ”message” polynomial will have the coefficients:

$$64, 212, 134, 86, 198, 198, 242, 194, 5, 118, 247, 38, 198, 66, 16, 236$$

Dividing it by the generator polynomial, we get a remainder with the coefficients:

$$215, 92, 247, 55, 155, 152, 59, 246, 87, 124$$

that we convert to binary bytes, giving us:

11010111	01011100	11110111	00110111	10011011
10011000	00111011	11110110	01010111	01111100

3.2.6 Interleaving

Now that we have computed the error correction codewords, we need to arrange them in a certain manner with the data codewords. The codewords go in the following order:

Data	Codeword 1	Codeword 2	Codeword 3	Codeword 4	Error correction	Codeword 1	Codeword 2	Codeword 3
Block 1	1	5	9	x	Block 1	15	19	23
Block 2	2	6	10	x	Block 2	16	20	24
Block 3	3	7	11	13	Block 3	17	21	25
Block 4	4	8	12	14	Block 4	18	22	26

In this example, B1 = 2, C1 = 3, B2 = 2, C2 = 4.

In our case, since we only have 1 data block, the error correction codewords are simply appended after the data codewords.

Similarly to step 3.2.4 (Data encoding), we need to pad the end result with a certain number of 0s before continuing. This number is given by the following table:

Version	Number of 0s to add
2 to 6	7
14 to 20	3
21 to 27	4
28 to 34	3

Our example code is a version 1 so we don’t need to add any 0.

3.2.7 Separators and finder patterns

Data has been encoded and now starts the placement phase, that is the creation of the black and white matrix.

First of all, the matrix' size (in number of modules) is given by the following formula:

$$(V - 1) * 4 + 21$$

where V is the version.

The ISO standard[4] also states that a 4-module wide margin (silence zone) must be respected all around the code. This allows scanners to easily identify and locate a QR-Code in an image.

The distinctive elements of QR-Codes are of course their three large corner squares. These are called "finder patterns" and are used by the reading device to find the code and correct the perspective. They also provide information on the rotation of the image and the width of individual modules. Finder patterns are 7x7 black squares, containing a 5x5 white square, encircling itself a 3x3 black square. They are put in the top-left, top-right and bottom-left corners of the matrix.

Additionally, they are separated from the rest of the code by a 1-module thick white line called a separator.

For our version 1 QR-Code, this steps yields the following 21x21 matrix:

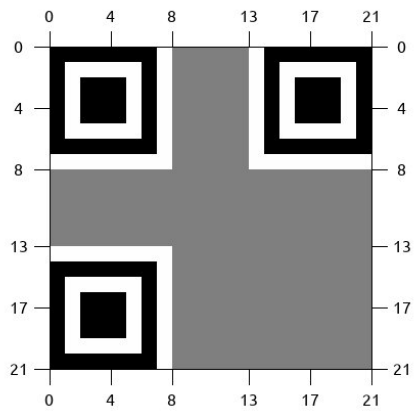


Figure 3.1: QR-Code example: separators and finder patterns

3.2.8 Alignment patterns

The next element to add are the alignment patterns. These are similar to finder patterns but are only 5x5. They are spread across the whole code and provide reference points for the scannnig device to improve reliability. The larger the code, the more alignment patterns are needed. Their positions depend on the version and are referenced in table D.4. This table lists all possible x and y coordinates for the patterns. This means that for version 2, the alignment patterns are located at (6,6), (6,18), (18,6) and (18,18). A pattern will only be present if the area it covers is still empty (i.e. it doesn't overlap with the separators and finder patterns).

Since our example is a version 1 QR-Code, no alignment pattern is needed (see figure 3.3 for version 10 code with alignment patterns).

3.2.9 Timing pattern

An additional element helping the scanner and improving readability is the timing pattern. It consists of a alternating black and white stripe joining the bottom-left and top-left finder pattern, and the top-left and top-right.

The pattern is aligned to the right and bottom borders of the top-left finder patterns, starting with white on the separators.

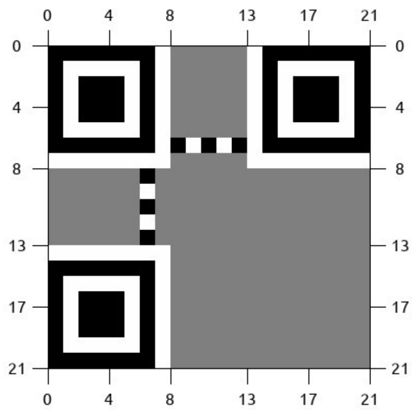


Figure 3.2: QR-Code example: timing patterns

3.2.10 Reserved area

Some areas of the matrix are also reserved for format information which will be added later. This corresponds to the modules around the top-left finder pattern, the modules on the right of the bottom-left pattern and those just below the top-right one.

Additionally, for versions greater than 6, a 3x6 zone on the left of the top-right finder pattern plus a 6x3 above the bottom-left one are reserved for version information.

Figure 3.3 shows an example of these reserved areas (in light gray) for a version 10 QR-Code (left) and for our example (right).

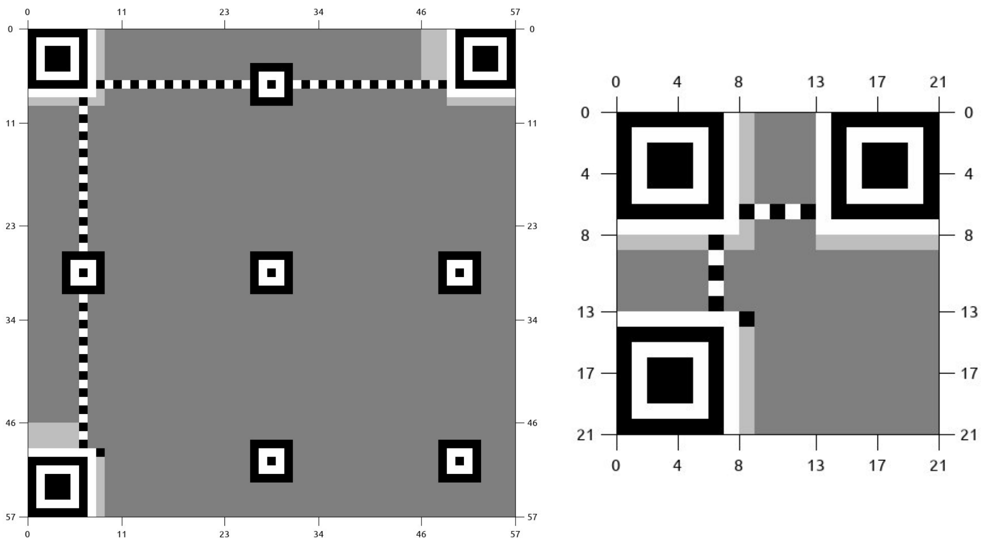


Figure 3.3: QR-Code example: reserved areas

A black module is also set next to the bottom-left finder pattern, on the right of its top-right corner. Its coordinates are the following: $\begin{cases} x = 8 \\ y = 4V + 9 \end{cases}$ where V is the version.

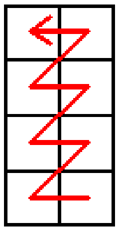
3.2.11 Data placement

The matrix is now ready to receive the data bit string. The placement is done in zigzags, starting from the bottom-right, going up. Each byte is placed in a 2 modules wide region in a staggered manner. When a pattern, separator or reserved area is encountered, the position is skipped and the process continues further.

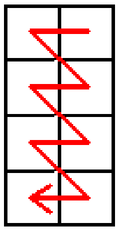
Figures 3.4a and 3.4b represent the way a byte is layed out when going up or down.

Figures 3.4c and 3.4d show how skipping and turning are processed.

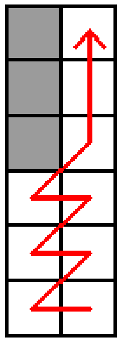
When encountering the vertical timing pattern, that column is entirely skipped and the placement continues on the next one. For each byte, the bits are layed from Most Significant bit (MSB) to Least Significant Bit (LSB).



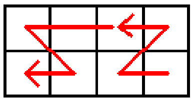
(a) Going up



(b) Going down



(c) Skipping reserved areas



(d) Turning at border

Figure 3.4: QR-Code byte placement

If the available space if not fully filled after placing the data, the rest is filled with 0s[3].

Our example QR-Code, once filled with data, looks like this:

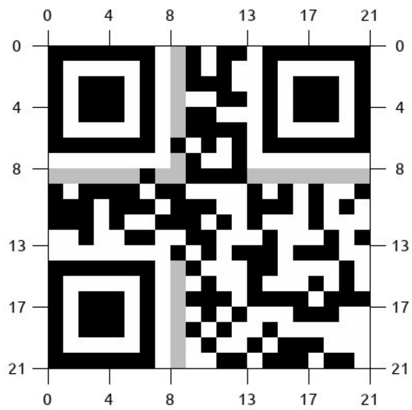


Figure 3.5: QR-Code example: data placement

[3] it shall be recalled that 0 means white and 1 means black

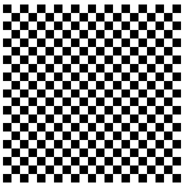
3.2.12 Masking

For optimal readability, it is important that certain patterns of modules don't appear inside the code. For example, there shouldn't be any shape resembling the finder or alignment patterns (i.e. modules with the ratio 1:1:3:1:1). Furthermore, a balanced amount of black compared to white is preferred for better decoding.

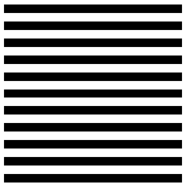
For this purpose, we need to apply a mask on the code, switching white for black modules and vice versa where it applies. QR-Codes have 8 different masks which can be used. Obviously, these are only applicable on the data area and should not modify the timing, finder and alignment patterns.

To choose one, we will apply them one after the other on our current QR-Code, and evaluate the resulting code, giving it a penalty score for each undesired feature. Then, the mask with the lowest score will be chosen.

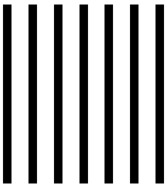
Figure 3.6 lists the different possible masks. A black mask module means that the corresponding data module needs to be inverted. The operator "//" is integer division.



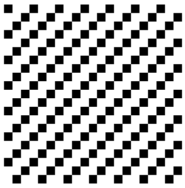
(a) (x+y) mod 2 = 0



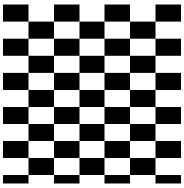
(b) y mod 2 = 0



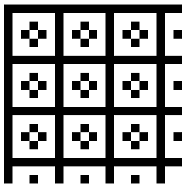
(c) (x) mod 3 = 0



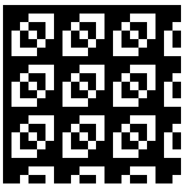
(d) (x+y) mod 3 = 0



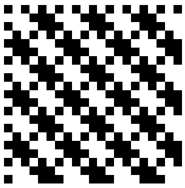
(e) (y//2+x//3) mod 2 = 0



(f) ((x*y) mod 2 + (x*y) mod 3) = 0



(g) ((x*y) mod 2 + (x*y) mod 3) mod 2 = 0



(h) ((x+y) mod 2 + (x*y) mod 3) mod 2 = 0

Figure 3.6: QR-Code masks

Evaluation

Evaluation of a mask is done thanks to 4 criteria.

They evaluate whether the code is easy to read or not. For example, criterion 3 gives a high penalty for every pattern with the same proportions as finder patterns to avoid confusion for the reading device.

Before applying a mask for evaluation, format and version information have do be added on the code, as described in sections 3.2.13 and 3.2.14.

1. 5+ consecutive modules of the same color:

   If a line or column of 5 or more modules of the same color is found in the code, a penalty score is added. For a strip of $5+i$ same colored modules, the penalty is worth $3+i$ points.

2. 2x2 blocks:

   Each 2x2 block of similar modules adds 3 points to the penalty score. Overlapping blocks are taken into account.

3. 1:1:3:1:1:4 patterns:

   For each pattern with the ratios 1:1:3:1:1:4 or 4:1:1:3:1:1, a penalty of 40 points is given. This criterion takes into account the 4-module wide margins all around the code[4]. This means there are at least 18 correspondances in every code.

4. Proportion of black and white modules:

   A penalty is attributed according to the deviation from a 50/50 distribution in black and white modules across the whole QR-Code. The calculation method is the following:

$$P = \lfloor 100 * B/(W * H) \rfloor$$

$$P_1 = P - P \text{ mod } 5$$

$$P_2 = P_1 + 5$$

$$S = \text{min}\left(\frac{|P_1 - 50|}{5}, \frac{|P_2 - 50|}{5}\right) * 10$$

where B is the total number of black modules, W and H are the width and height of the QR-Code, and S is the penalty score given for this criterion.

Applying this to our QR-Code, we can determine that the best mask is mask 3.6f with a score of 442. Figures 3.7a to 3.7d detail the penalties for each criterion.

[4] see subsection 3.2.7

(a) QR-Code example: mask evaluation (1)

(b) QR-Code example: mask evaluation (2)

(c) QR-Code example: mask evaluation (3)

(d) QR-Code example: mask evaluation (4)

Figure 3.7: QR-Code example: mask evaluation

3.2.13 Format information

The last step to complete a fully functional QR-Code is to add the format information, and version information for versions bigger than 6.

First we need to create a format string containing the level of error correction and the mask used. The correction level is encoded on 2 bits as follows:

Level	Value (bin)
L	01
M	00
Q	11
H	10

Table 3.3: QR-Code error correction level indicator

Then the mask id is converted to a 3-bit binary number and appended to the error correction level indicator.

The string is padded by an additional 10 0s to make it 15 bits long.

Similarly to what has been done with data previously, the format string is complemented with error correction bits, this time using Bose-Chaudhuri-Hocquenghem (BCH) codes. The principle is similar to the Reed-Solomon algorithm in that a message polynomial is divided by a generator polynomial. To create the message polynomial, each bit of the format string represents the coefficient of a term. The same applies for the generator polynomial, which is always derived from the binary number `10100110111`. The generator polynomial is thus

$$x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1$$

The division's remainder is then padded on the left with 0s to make it 10 bits long.

The final bit string is constructed by concatenating the format string with the error correction bits, and XORing[5] the result with the mask string `101010000010010`. This mask ensures the final format string is not made of only 0s.

Once calculated, the bit string is layed out in the reserved strips around the finder patterns, as shown in figure 3.8 (0 being the LSB and 14 the MSB)

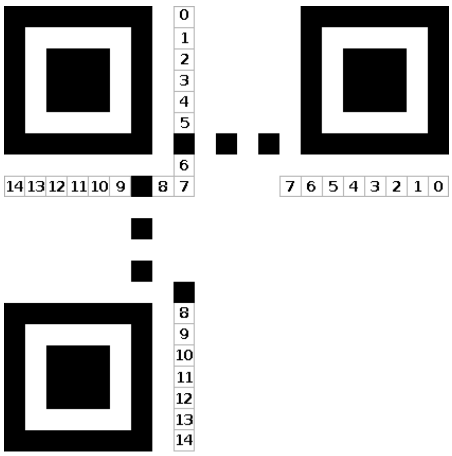


Figure 3.8: QR-Code format information string layout

[5]With the binary XOR operator

It is to be noted that format information appears twice, since its decoding is essential for reading the whole code.

In our case, the error correction indicator for level M is 00 and we used the mask with id 2, so our format string is `000100000000000`. Dividing it by the generator polynomial yields the remainder `1001101110`.

Adding it to the format string and XORing it with the mask string, we get `101111001111100`.

These bits are then put in the reserved areas of the matrix, making figure 3.9

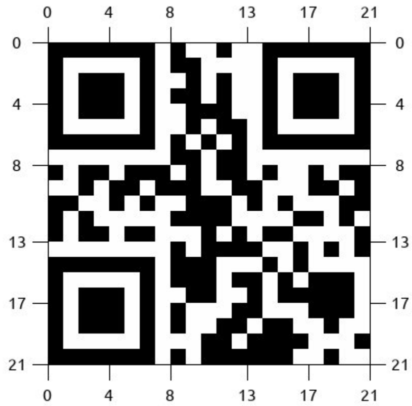


Figure 3.9: QR-Code example: format information

Our QR-Code is now fully finished and can be scanned.

3.2.14 Version information

For QR-Codes of version 7 and bigger, additional data is added to state the code's version.

To generate the version string, first convert the version to its 6 bit binary representation. Then append the remainder of the division by the generator polynomial

$$x^{12} + x^{11} + x^{10} + x^9 + x^8 + x^5 + x^2 + 1$$

padded on the left to 12 bits, following the same methods as for format information.

This string is then put in the two reserved 6x3 and 3x6 rectangles. The LSB is placed in the top-left corner of the rectangles. For the top-right area, the string goes down, then to the right. For the bottom-left rectangle, the string goes to the right, then down.

3.3 Application in Python

In this section, we will look at my Python QR-Code generator implementation. For the sake of brevity, only some specific parts of the program will be commented.

3.3.1 Python features

The script takes advantage of several Python-specific features.

Dunder methods

The most important is the "dunder methods", short for "double underscore methods". These are special overridable methods used for builtin behaviors.
For example, the `__add__`, `__sub__`, `__mul__`, `__truediv__` and `__pow__` methods respectively define the behavior of the "+", "-", "*", "/" and "**" operators.

This is particulary useful to create the Galois field's arithmetic used for QR-Codes. For example, multiplication is defined by this method:

```
def __mul__(self, n):
    if self.val == 0 or n.val == 0:
        return GF(0)

    return GF.EXP[GF.LOG[self.val].val + GF.LOG[n.val].val].copy()
```

where `val` is the element's value and `GF.LOG` and `GF.EXP` are arrays containing the values of exponents and logarithms for the field (see subsection 3.3.2).

Anonymous functions

Anonymous, or lambda, functions are short unnamed functions. They are often used for very basic operations. In our case, they are utilized for masks.

For example, the first mask is defined as `lambda x,y: (x+y)%2 == 0`, a function taking two arguments x and y and returning whether the coordinates should be masked or not.

3.3.2 Precomputed data

Some values related to the creation of QR-Codes are precomputed, such as the capacities for each data type or the number of error correction codewords, as determining them is done through reverse engineering and no simple direct formula can be established. These values are stored in text files (`error_correction.txt` and `qr_versions.txt`) and loaded into tables at the beginning of the scripts.

Regarding Galois fields, all powers and logs are also calculated beforehand, for the sake of ease of use, using the following loops:

```
class GF:
    def __init__(self, val):
        self.val = val
    ...

GF.EXP = [GF(0)]*512
GF.LOG = [GF(0)]*256
value = 1
for exponent in range(255):
    GF.LOG[value] = GF(exponent)
    GF.EXP[exponent] = GF(value)
    value = ((value << 1) ^ 285) if value > 127 else value << 1

for i in range(255, 512):
    GF.EXP[i] = GF.EXP[i-255].copy()
```

Credits for this method goes to *Reed–Solomon codes for coders — Wikiversity,*[12, Multiplication with logarithms]

3.3.3 Data placement

One of the challenges to overcome was the data placement phase. To avoid lengthening this particular part, mathematical tricks are used.

For visual aid, see figure 3.4 in subsection 3.2.11.

Before starting, the data bit string which will be placed is stored in a string variable named "`self.final_data_bits`". The position is set to the lower-right corner of the matrix. The matrix (`self.matrix`) is a 2D array in which -1 indicates a free module.

A variable named `dir_` is also set to -1 and is responsible to keep track wether we are going up or down. A variable `i` initialized to 0 will hold the index of the current bit to be placed. The variable `zigzag` manages the zigzag pattern.

```
dir_ = -1  #-1 = up | 1 = down
x, y = size-1, size-1
i = 0
zigzag = 0
```

```
5
6  while x >= 0:
7      if self.matrix[y,x] == -1:
8          self.matrix[y,x] = self.final_data_bits[i]
9          i += 1
10
11     if ((dir_+1)/2 + zigzag)%2 == 0:
12         x -= 1
13
14     else:
15         y += dir_
16         x += 1
17
18     if y == -1 or y == size:
19         dir_ = -dir_
20         y += dir_
21         x -= 2
22
23     else:
24         zigzag = 1-zigzag
25
26     if x == 6:
27         x -= 1
```

The algorithm runs until it reaches the left side (line 6).

For each loop, if the module is free, the current bit is placed and `i` is incremented.

If we are going up and `zigzag` equals 0, or if we are going down and zigzag equals 1, then we move to the left (line 11-12).
Otherwise, we move forward in the current direction and one module to the right.

If we reach the top or bottom side (current position is outside of the matrix), the direction is flipped, we come back one step and move to the left.

Lines 26-27 make the placement entirely skip column 6, which is where the vertical timing pattern is located.

Table 3.4 shows the evolution of the different variables during placement.

x	y	dir_	zigzag
20	20	-1	0
19	20	-1	1
20	19	-1	0
19	19	-1	1
20	18	-1	0
...	...	...	...
20	0	-1	0
19	0	-1	1
18	0	1	1
17	0	1	0
18	1	1	1

Table 3.4: QR-Code data placement algorithm

3.3.4 Mask evaluation

Mask evaluation is quite straight-forward especially for criteria 1, 2 and 4 (see section 3.2.12)

Criterion n° 3 is a bit more complex. To keep track of the patterns encountered in each row (or column), a `History` object is used. This object holds a list of widths of the different color zones.

Chapter 4

Error detection and correction

This chapter introduces two methods to create self-correcting messages: Hamming codes and Reed-Solomon codes. The former is based on parity bits while the latter takes advantage of advanced mathematical properties of modular arithmetic and polynomials.

4.1 Hamming Codes

When working with binary data, one way of checking if a received message is corrupted or not is to add a parity bit. The parity of a binary number is even if it has an even number of 1s and odd otherwise. A parity check bit is added such that the total parity of the number is even, i.e. 0 if it is already even, 1 otherwise.

					parity bit	
bit 1	bit 2	bit 3	bit 4	bit 5	bit 6	parity: even
1	1	0	0	1	1	

With this, a single bit error (that is, one bit is wrong) is easy to detect because the parity of the message becomes odd.

bit 1	bit 2	bit 3	bit 4	bit 5	bit 6	
1	1	**1**	0	1	1	parity: odd

However, a single parity bit doesn't provide enough information to allow locating the error or detecting multiple errors, because an even number of errors would keep an even parity overall.

bit 1	bit 2	bit 3	bit 4	bit 5	bit 6	
1	1	**1**	0	**0**	1	parity: even

Hamming codes are a kind of parity check codes. Instead of using only one parity bit however, they include several so that locating becomes possible, as well as detecting (not always) multiple errors.

When creating a Hamming code from a message, data first has to be split into blocks of a given size. For each block a certain number of parity bits is assigned. These two variables (blocksize and number of parity bits) determine the type of Hamming code. For example a Hamming code with 3 parity bits will form 7-bit blocks, meaning each block can hold 4 data bits. It can thus be called Hamming(7, 4).

Smaller blocksizes allow more errors to be corrected, because each block can correct one error, but have a lower data density[1]. On the other hand, larger blocksizes allow less errors to be corrected but have a higher data density.

Hamming codes are created in such a way that when a bit is flipped, the parity bits indicate exactly where the error occured. For that, each position in the code which is a power of two is a parity bits. Then, each parity bit covers the parity of all bits at positions containing its power in their binary representation. For example, the parity bit at position 4 (0b100) covers bits 5 (0b101), 6 (0b110), 7 (0b111), 12 (0b1100), 13 (0b1101), 14 (0b1110), 15 (0b1111), ...

Table 4.1 taken from *Hamming code — Wikipedia, The Free Encyclopedia*[9] offers a good visual representation of this structure:

Bit position		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Encoded data bits		p1	p2	d1	p4	d2	d3	d4	p8	d5	d6	d7	d8	d9	d10	d11
Parity bit coverage	p1	✓		✓		✓		✓		✓		✓		✓		✓
	p2		✓	✓			✓	✓			✓	✓			✓	✓
	p4				✓	✓	✓	✓					✓	✓	✓	✓
	p8								✓	✓	✓	✓	✓	✓	✓	✓

Table 4.1: Hamming code structure

Here we can see that each data bit (d1, d2, d3, ...) is covered by a unique set of parity bits.

[1] data density is the ratio of data bits over blocksize

Let's create a Hamming(15, 11) code for the message `11101100010`. The first step is to lay out the bits in table 4.1 like so:

Bit position		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Encoded data bits		p1	p2	1	p4	1	1	0	p8	1	1	0	0	0	1	0
Parity bit coverage	p1	-		✓		✓		✗		✓		✗		✗		✗
	p2		-	✓			✓	✗			✓	✗			✓	✗
	p4				-	✓	✓	✗					✗	✗	✓	✗
	p8								-	✓	✓	✗	✗	✗	✓	✗

Parity bit	Covered 1s	Parity of covered bits	Value
p1	3	odd	1
p2	4	even	0
p4	3	odd	1
p8	3	odd	1

Table 4.2: Hamming code example

Placing the parity bits in their relevant positions, we get the hamming code `101111011100010`.

To illustrate the decoding process, let's alter bit 11 and change it to a 1. Now, recalculating the parity bits and comparing the results with the received message, we can find the location of the error.

Bit position		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Received data bits		1	0	1	1	1	1	0	1	1	1	**1**	0	0	1	0
Parity bit coverage	p1	-		✓		✓		✗		✓		✓		✗		✗
	p2		-	✓			✓	✗			✓	✓			✓	✗
	p4				-	✓	✓	✗					✗	✗	✓	✗
	p8								-	✓	✓	✓	✗	✗	✓	✗

Parity bit	Covered 1s	Parity of covered bits	Value	Received value
p1	4	even	0	1
p2	5	odd	1	0
p4	3	odd	1	1
p8	4	even	0	1

Table 4.3: Hamming code example decoding

The difference (XOR) between columns ”Value” and ”Received value” forms the binary number `0b1101` = 11, the location of the error.

4.2 Reed-Solomon algorithm

The Reed-Solomon algorithm is a mathematical process allowing the decoding of a partially corrupted message. It is used in many domains for its strength and reliability, such as for spatial communication, CD/DVD players, some television broadcasts and QR-Codes.

Reed-Solomon codes were developed by Irving S. Reed and Gustave Solomon in 1960 [7]

As they rely on abstract and complex mathematical concepts, this section will not go in specific details about the actual decoding process. For curious readers, see *Reed Solomon Encoding - Computerphile*[1] (presentation, general concept), *Reed–Solomon error correction — Wikipedia, The Free Encyclopedia*[11] (in depth article) and *Tutorial on Reed-Solomon Error Correction Coding*[2] (complete manual and explanations)

4.2.1 Error detection

Before considering error correction, it is necessary to talk about error detection. Several methods have been developed for this purpose.

The most basic, as seen in section 4.1, is a parity check. It consists of appending one or more ”parity bits” to a binary message, such that the overall parity is known. For example, let table 4.4 be our raw message

0	1	0	0	0	0	1	1

Table 4.4: Error detection: raw message

The parity of this byte is odd – because there are 3 1s – so an additional 1 is added to the end.

In this way, if the message is corrupted – by 1 bit maximum – it becomes even, and we know there is an error. Now obviously it doesn’t provide any information on the exact location of the error in the message and can’t detect an even number of errors.

However, this principle can be extended to include a parity bit for every byte. If represented as a table in which each row is a byte, it can also include parity bits for each column. Table 4.5 is an example of such usage of parity bits. This implementation is even able to correct a single error as the row and column would be odd.

	bit 0	bit 1	bit 2	bit 3	bit 4	bit 5	bit 6	bit 7	parity
byte 0	0	1	0	0	0	0	1	1	1
byte 1	0	1	1	0	1	1	1	1	0
byte 2	0	1	1	0	0	1	0	0	1
byte 3	0	1	1	0	0	1	0	1	0
byte 4	0	1	1	1	0	0	1	1	1
parity	0	1	0	1	1	1	1	0	1

Table 4.5: Error detection: bytes table parity

Such codes however don't provide enough error correction capability - at most 1 bit. Other methods, like the previously explained Hamming codes, allow a more efficient use of parity bits, increasing to some degree the number of fixable errors.

Reed-Solomon codes use properties of polynomials and modular arithmetic to produce more efficient and more robust correction data.

4.2.2 Binary to polynomials

Instead of working directly with binary data, Reed-Solomon codes treat messages as polynomials. These are formed from binary data as follows: each byte is converted to a decimal integer, representing coefficients of the polynomial.

For example:

raw binary	01000011	01101111	01100100	01100101	01110011
decimal	67	111	100	101	115
polynomial	$67x^4 + 111x^3 + 100x^2 + 101x + 115$				

4.2.3 Galois Fields

Since the main applications of Reed-Solomon codes are related to digital devices, it is relevant to use bits and bytes. As such, all calculations are performed in a Galois field. A Galois field is basicly a finite set of numbers on which arithmetic operations results in numbers of the set. In the case of QR-Codes, G(256) – a Galois field of the integers 0 to 255 incl. – is used. This means every operation between numbers results in a value between 0 and 255 incl., which is an eight-bit positive integer. In this field, addition and subtraction are equivalent and defined as the binary XOR operation. For example:

$$17 + 13 = 17 - 13 = 28$$
$$\Leftrightarrow 0\text{b}10001 \oplus 0\text{b}1101 = 0\text{b}1101 \oplus 0\text{b}10001 = 0\text{b}11100$$

Multiplication is more complex though. One property of this Galois field is that every number can be represented as a power of two, XOR 285. For example:

$$
\begin{aligned}
2^{17} =& (\underbrace{2^8}_{256} \oplus 285) * 2^9 = 29 * 2^9 \\
=& [\underbrace{(29 * 2^4)}_{464} \oplus 285] * 2^5 = 205 * 2^5 \\
=& [\underbrace{(205 * 2)}_{410} \oplus 285] * 2^4 = 135 * 2^4 \\
=& [\underbrace{(135 * 2)}_{270} \oplus 285] * 2^3 = 19 * 2^3 \\
=& 152 \\
\Rightarrow & \ \mathsf{exp}_2(17) = 152 \\
\Rightarrow & \ \mathsf{log}_2(152) = 17
\end{aligned}
$$

To multiply two numbers a and b in the Galois field:[2]

$$a * b = \mathsf{exp}(\mathsf{log}(a) + \mathsf{log}(b))$$

which also works in regular arithmetic (in $\mathbb{N}_+^*$).

Division works similarly, but because there are no negative or fractional number in the field, the exponent is kept in the range 0-255 incl. like so:

$$\frac{a}{b} = \mathsf{exp}([\mathsf{log}(a) - \mathsf{log}(b) + 255] \ mod \ 255)$$

And powers too:

$$a^b = \mathsf{exp}([\mathsf{log}(a) * b] \ mod \ 255)$$

4.2.4 Generating error correction

To create Reed-Solomon error correction bytes, a generator polynomial $g(x)$ is needed. This polynomial is created using equation 4.1:

$$g(x) = \prod_{i=0}^{d-1} (x + 2^i) \tag{4.1}$$

where d is one more than the degree of the polynomial, equivalent to the number of error correction bytes.

Let $m(x)$ be our message polynomial (see subsection 4.2.2) and $g(x)$ the generator polynomial. The error correction polynomial $E_c(x)$ is then the remainder of the long polynomial division $m(x)/g(x)$.

[2] from now on, exp and log are assumed to be base 2

Let's illustrate this by creating error correction for the string "Codes". In UTF-8, the message bytes are 67, 111, 100, 101, 115, thus $m(x) = 67x^4 + 111x^3 + 100x^2 + 101x + 115$. We will take the generator polynomial of degree 4, that is:

$$g(x) = (x+1) * (x+2) * (x+4) * (x+8) = x^4 + 15x^3 + 54x^2 + 120x + 64$$

And thus (reminder that addition and subtraction in the galois field is the binary XOR operation):

$$\begin{array}{rrrrrrrrr|rrrrr}
67 & 111 & 100 & 101 & 115 & 0 & 0 & 0 & 0 & 1 & 15 & 54 & 120 & 64 \\ \cline{10-14}
-67 & -246 & -91 & -227 & -13 & & & & & 67 & 153 & 107 & 43 & 8 \\ \cline{1-5}
 & 153 & 63 & 134 & 126 & & & & & & & & & \\
 & -153 & -84 & -222 & -154 & -137 & & & & & & & & \\ \cline{2-6}
 & & 107 & 88 & 228 & 137 & & & & & & & & \\
 & & -107 & -115 & -120 & -191 & -223 & & & & & & & \\ \cline{3-7}
 & & & 43 & 156 & 54 & 223 & & & & & & & \\
 & & & -43 & -148 & -121 & -212 & -18 & & & & & & \\ \cline{4-8}
 & & & & 8 & 79 & 11 & 18 & & & & & & \\
 & & & & -8 & -120 & -173 & -231 & -58 & & & & & \\ \cline{5-9}
 & & & & & 55 & 166 & 245 & 58 & & & & &
\end{array}$$

$$\Rightarrow E_c(x) = 55x^3 + 166x^2 + 245x + 58$$

Details of the first step:

$$\begin{aligned}
67 * 1 &= \mathsf{exp}(\mathsf{log}(67) + \mathsf{log}(1)) = \mathsf{exp}(98 + 0) = \mathsf{exp}(98) = 67 \\
67 * 15 &= \mathsf{exp}(\mathsf{log}(67) + \mathsf{log}(15)) = \mathsf{exp}(98 + 75) = \mathsf{exp}(41) = 246 \\
67 * 54 &= \mathsf{exp}(\mathsf{log}(67) + \mathsf{log}(54)) = \mathsf{exp}(98 + 249) = \mathsf{exp}(155) = 91 \\
67 * 120 &= \mathsf{exp}(\mathsf{log}(67) + \mathsf{log}(120)) = \mathsf{exp}(98 + 78) = \mathsf{exp}(44) = 227 \\
67 * 64 &= \mathsf{exp}(\mathsf{log}(67) + \mathsf{log}(64)) = \mathsf{exp}(98 + 6) = \mathsf{exp}(100) = 13
\end{aligned}$$

Then, to communicate our message, $E_c(x)$ is converted to binary and appended to our raw message data, in our case, the final message would be: 67, 111, 100, 101, 115, 55, 166, 245, 58.

This is the actual data sent by a device, or in the case of QR-Codes, the actual data encoded on the symbol. Let it be a polynomial named $s(x)$ (for sent data).

Unfortunately, this is not always what is received by the recipient (or read by the scanner). Some interference may happen during transmission and data may be altered. Let the received data be the polynomial $r(x) = s(x) + e(x)$ (where $e(x)$ is the error polynomial).

In the next section, we will outline the main steps and basic mathematical principles required for error correction and detection through the Reed-Solomon algorithm.

4.2.5 Detecting and correcting errors

The first step to locating potential errors in a received Reed-Solomon code is to calculate its ”syndrome polynomial” $S(x)$. The coefficient of the i^{th} degree term of this polynomial is the value of $r(2^i)$ (the degree of $S(x)$ is equal to the number of error correction bytes minus 1, in our case 2). This means:

$$S(x) = \sum_{i=0}^{d-1} r(2^i) * x^i$$

To illustrate the algorithm, we will take

$$r(x) = 67x^8 + 111x^7 + \mathbf{110}x^6 + 101x^5 + 115x^4 + \mathbf{50}x^3 + 166x^2 + 245x + 58$$
$$\Rightarrow e(x) = 10x^6 + 5x^3$$

Thus,

$$S(x) = 253x^3 + 252x^2 + 146x + 15$$

Reed-Solomon codes provide a very useful mathematical property. In fact, if $s(x) = r(x) \Rightarrow e(x) = 0$, then $S(x) = 0$, enabling a fast return if there is no corruption.

In the case where $S(x) \neq 0$, we need to compute two other polynomials, the locator and evaluator polynomials. The former helps determine positions of errors whilst the latter is used to find the magnitude of each error, that is, the difference with the real value.

These can be found with the help of the euclidean algorithm. The exact methods used will not be described here as the mathematical implications behind them are much above the level of this work, but there functioning and alternatives are well documented in *Tutorial on Reed-Solomon Error Correction Coding*[2] (from p.65, section 4.3.1).

From our example, we would get the following polynomials:

$$E_{locator}(x) = 58x^2 + 72x + 1$$

$$E_{evalutor}(x) = 13x + 15$$

Locator polynomial Once the locator polynomial has been computed, it can be used to get the precise position of each error, as long as the number of errors is not greater than the correction capacity.

The error location polynomial[3] is first calculated from the locator polynomial using Chien search (not described here), a ”fast algorithm for determining roots of polynomials defined over a finite field”[8].

In this polynomial, each coefficient’s log (in the Galois field) is the byte index of an error in the received message (starting from the end) – or degree of a wrong coefficient in $r(x)$.

Continuing the example, we obtain:

$$E_{location}(x) = 64x + 8$$

Evaluator polynomial Using the error location and evaluator polynomial in Forney’s algorithm, it is possible to find the magnitude of each error, that is the coefficients of $e(x)$.

Our result:

$$E_{mag}(x) = 10x + 5$$

Correction We now have all the information needed to correct the received message. For that, we need to add the magnitudes to their corresponding locations. Again, the locations are the logarithms of each coefficient in the error location polynomial and magnitudes are the coefficients of E_{mag}.

Our example has two errors, since both $E_{location}$ and E_{mag} are second degree polynomials. For the first error, we add 10 to r_6 (6 being log(64)). For the second error, we add 5 to r_3 (3 being log(8)).

We can finally recover the original message: 67, 111, 100, 101, 115, 55, 166, 245, 58.

[3] not to be confused with the error locator polynomial

Chapter 5

Custom code

In this chapter, we will create a new type of 2D code, based on concepts discussed in this work: the Lycacode.

Some design choices have been made for ease of use and others for aesthetic purposes, each explained in their relevant section.

The basic format is in the form of a trefoil cross, the blazon of Saint-Maurice and of the Collège de l'Abbaye.

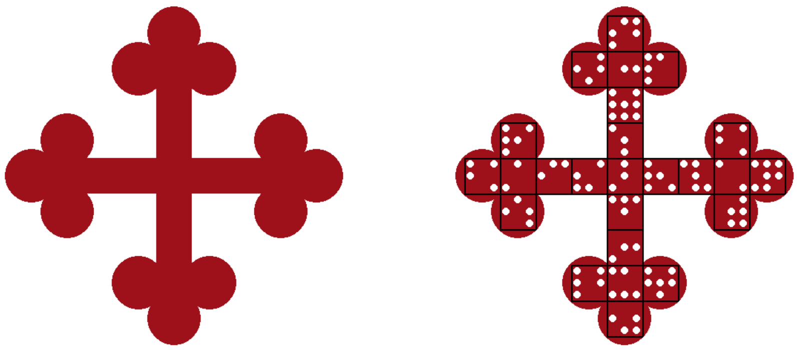


Figure 5.1: Lycacode: trefoil cross and squares

This cross is split into 25 squares which hold data. Each square is made of a 3x3 grid of dots and blanks, representing bits. The right of figure 5.1 shows how data (white dots) is put in the squares (highlighted in black)[1]. A dot represents a 1 while the absence of one is a 0.

[1] the black squares are simply visual aids and are not part of the final code

The decision of using a grid-like pattern is convient for data placement as well as for reading the code.

The central square is reserved for the orientation pattern, as described in subsection 5.3.3. Additionally, the three top- and bottom-most dots are saved for the mask id (see subsection 5.3.4).

5.1 Encoding

This code can work in one of four modes:

0. Person
1. Location
2. Link
3. Text

5.1.1 Person - mode 0

In mode 0, the code represents a person from the school. It can either be a student (type 0), a teacher (type 1) or someone else (type 2), such as the cleaning staff or caretaker. Table 5.1 lists the different values encoded by each type and their corresponding bit size. Column ”bit size” is the number of bits the value is encoded on. These have been chosen to use as few bits as possible to leave room for possible future additional data.

Type	Value	Bit size
Student	type	2
	id	20
	year	3
	class	4
	initials	10
Teacher	type	2
	id	20
Other	type	2
	id	20

Table 5.1: Lycacode: person mode - values

For students

`year` is the year number. DUBS is represented by the value 0.
`class` is the index of the class letter in the alphabet, starting from 0. For example, D is 3.
`initials` represent the initial of the firstname and that of the lastname, each as 5 bit numbers. The value of each letter is their index in the alphabet, starting from 0.

Example

type	id	year	class	initials
Student	16048	5	D	LH

Bits: 00 / 00 / 00000011111010110000 / 101 / 0011 / 01011 / 00111

5.1.2 Location - mode 1

In mode 1, the code represents a location in the school. The section is encoded on 3 bits according to the following table:

Section	Value
A	0
B	1
C	2
D	3
Boarding school	4
”Bateau”	5
Sports alley	6
Football fields	7

Table 5.2: Lycacode: location mode - sections

Additionally the room number (or other id) is encoded on 9 bits.

Example

section	room
4	209

Bits: 01 / 100 / 011010001

5.1.3 Link - mode 2

In mode 2, the code represents a URL. The actual URLs are stored in a database and only the id is saved on the code as a 32 bit number. Scanners then fetch the URL info from the server's database.

5.1.4 Text - mode 3

In mode 3, the code represents normal text. Text data is simply converted to UTF-8. The number of encoded characters is first encoded on 4 bits and added before text data. Due to its limited capacity, a Lycacode can only store up to 14 characters.

Example

length	text
4	Lyca

Bits: 11 / 0100 / 01001100 / 01111001 / 01100011 / 01100001

5.2 Error correction

It goes without saying that this code uses some kind of error correction. To keep it simple enough, Hamming(7, 4) codes have been chosen to fulfil this role. Encoded data is first padded to the maximum number of data bits M:

$$M = T * R$$

where T is the total number of bits which can be encoded on the cross and R is the ratio of data bits over blocksize (here, $R = \frac{4}{7}$). T can be calculated as follows:

$$T = \underbrace{3^2}_{\text{number of dots in a square}} * 24 - \underbrace{6}_{\text{mask id}}$$

5.3 Example

Let's create a Lycacode to illustrate. We will make a student code using the values from the example in subsection 5.1.1.

5.3.1 Data encoding

Table 5.3 lists all values to encode and their respective binary strings.

Property	Value	Binary
Mode	0	00
Type	0 (=student)	00
Id	16048	00000011111010110000
Year	5	101
Class	3 (=D)	0011
Initials	LH	01011 00111

Table 5.3: Lycacode: example values

The raw data bit string is thus:

$$\underbrace{00}_{\text{mode}}\underbrace{00}_{\text{type}}\underbrace{00000011111010110000}_{\text{id}}\underbrace{101}_{\text{year}}\underbrace{0011}_{\text{class}}\underbrace{0101100111}_{\text{initials}}$$

We then need to pad it to fill the remaining free bits. First we pad with zeros to the nearest multiple of 4 (data bits per block). Then we fill the rest with a pattern of consecutive binary numbers[2], like this:

$$01101110010111011110001001101010111100110111101111...$$

This pattern has the sole purpose of adding pseudo-random data so that there is data on the whole code. This is only an aesthetic choice.

[2]the pattern is the series of natural numbers in binary starting from 0, e.g. 0, 1, 10, 11, 100, ...

5.3.2 Hamming codes

Finally we construct the Hamming codes:

	1	2	3	4	5	6	7
Group 1	_	_	0	_	0	0	0
Group 2	_	_	0	_	0	0	0
Group 3	_	_	0	_	0	1	1
Group 4	_	_	1	_	1	1	0
Group 5	_	_	1	_	0	1	1
Group 6	_	_	0	_	0	0	0
Group 7	_	_	1	_	0	1	0
Group 8	_	_	0	_	1	1	0
Group 9	_	_	1	_	0	1	1
Group 10	_	_	0	_	0	1	1
Group 11	_	_	1	_	0	0	0
Group 12	_	_	0	_	1	1	0
Group 13	_	_	1	_	1	1	0
Group 14	_	_	0	_	1	0	1
Group 15	_	_	1	_	1	0	1
Group 16	_	_	1	_	1	1	0
Group 17	_	_	0	_	0	1	0
Group 18	_	_	0	_	1	1	0
Group 19	_	_	1	_	0	1	0
Group 20	_	_	1	_	1	1	1
Group 21	_	_	0	_	0	1	1
Group 22	_	_	0	_	1	1	1
Group 23	_	_	1	_	0	1	1
Group 24	_	_	1	_	1	1	0
Group 25	_	_	0	_	0	0	1
Group 26	_	_	0	_	0	0	1
Group 27	_	_	1	_	0	0	1
Group 28	_	_	0	_	1	0	0
Group 29	_	_	1	_	1	1	0
Group 30	_	_	1	_	0	0	1

	1	2	3	4	5	6	7
Group 1	0	0	0	0	0	0	0
Group 2	0	0	0	0	0	0	0
Group 3	1	0	0	0	0	1	1
Group 4	0	0	1	0	1	1	0
Group 5	0	1	1	0	0	1	1
Group 6	0	0	0	0	0	0	0
Group 7	1	0	1	1	0	1	0
Group 8	1	1	0	0	1	1	0
Group 9	0	1	1	0	0	1	1
Group 10	1	0	0	0	0	1	1
Group 11	1	1	1	0	0	0	0
Group 12	1	1	0	0	1	1	0
Group 13	0	0	1	0	1	1	0
Group 14	0	1	0	0	1	0	1
Group 15	1	0	1	0	1	0	1
Group 16	0	0	1	0	1	1	0
Group 17	0	1	0	1	0	1	0
Group 18	1	1	0	0	1	1	0
Group 19	1	0	1	1	0	1	0
Group 20	1	1	1	1	1	1	1
Group 21	1	0	0	0	0	1	1
Group 22	0	0	0	1	1	1	1
Group 23	0	1	1	0	0	1	1
Group 24	0	0	1	0	1	1	0
Group 25	1	1	0	1	0	0	1
Group 26	1	1	0	1	0	0	1
Group 27	0	0	1	1	0	0	1
Group 28	1	0	0	1	1	0	0
Group 29	0	0	1	0	1	1	0
Group 30	0	0	1	1	0	0	1

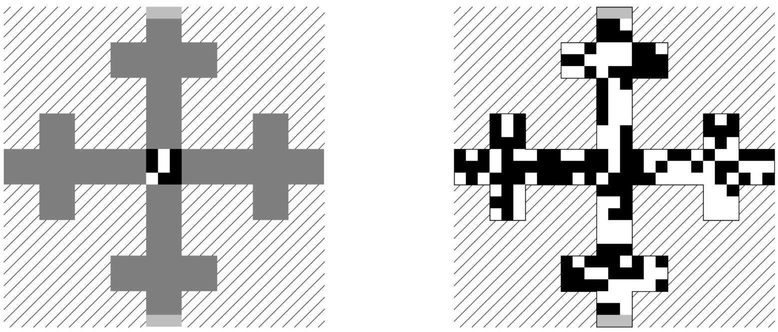
Table 5.4: Lycacode: example hamming codes

5.3.3 Laying out data

The matrix layout is shown in figure 5.2a. Notice the center square; it is used for rotation and mirror image detection. The middle pattern has to be asymmetrical both in reflection and rotation. Here, the top dot helps determine rotation, while the left one is used to check whether the code is mirrored or not. The central dot indicates that this is a Lycacode. Indeed, another type of code, Mini Lycacodes, has been created. Those don't have this dot, signaling that they are Mini Lycacodes.[3]

The top and bottom gray areas are reserved for the mask id as explained later. Also note that white means 1 and black 0.

Starting from the top left going in reading direction, the bits are layed out in the free areas. As for QR-Codes, the first bit of each group is first layed, then the second, the third and so on. Figure 5.2b shows the result of this step. The interleaving process allow a division of data in such a way that if a portion of the code is unreadable, the errors are distributed accross multiple data blocks, increasing the chance of recovery (since each block can only correct one bit).



(a) Empty (b) With data

Figure 5.2: Lycacode layout

5.3.4 Mask

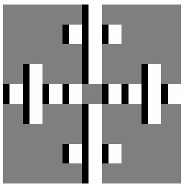
As a last step, a mask is applied. The 8 masks are described in figure 5.3. The best fitting one is selected based upon similar criteria as for QR-Codes[4]. Once applied to the data bits, the mask's id is encoded on the 3 reserved bits at the top and bottom of the code.

The purpose of masking in this context is purely aesthetical. It is a mean to avoid unpleasant

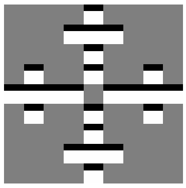
[3] Mini Lycacodes are not described here but are implemented in Python in the files `lycacode_gen_mini.py` and `lycacode_scanner_mini.py`

[4] the exact criteria are defined in the python script `lycacode_gen.py`

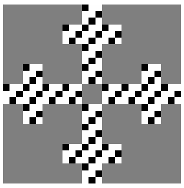
visual patterns in the final code.



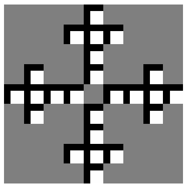
(a) x mod 3 = 0



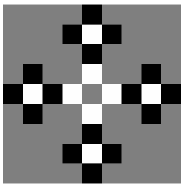
(b) y mod 3 = 0



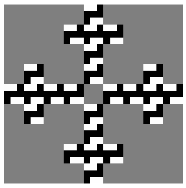
(c) (x+y) mod 3 = 0



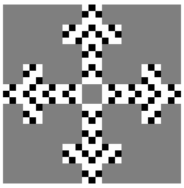
(d) (x mod 3)*(y mod 3) = 0



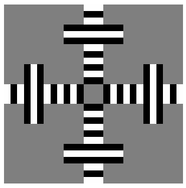
(e) (y//3+x//3) mod 2 = 0



(f) [(y mod 3)-1]*[(x mod 3)-(y mod 3)-2]*[(y mod 3)-(x mod 3)-2] = 0



(g) (|13-x|+|13-y|) mod 3 = 1



(h) [1-(x mod 2) + max(0, |13-y|-|13-x|)] * [1-(y mod 2) + max(0,|13-x|-|13-y|)] = 0

Figure 5.3: Lycacode masks

Note: ”//” is integer division

For our example, the best mask is mask 3. The final binary matrix is shown in figure 5.4.

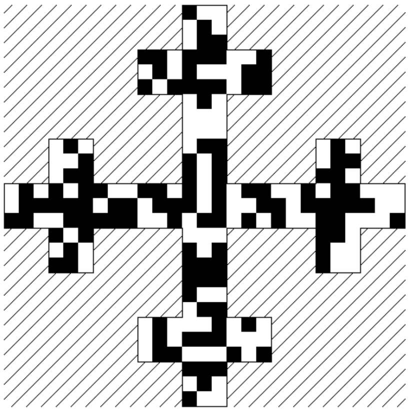


Figure 5.4: Lycacode example: masked matrix

Finally, the matrix is converted to white dots (for 1s) on the red trefoil cross shown in figure 5.1, giving the final code in figure 5.5.

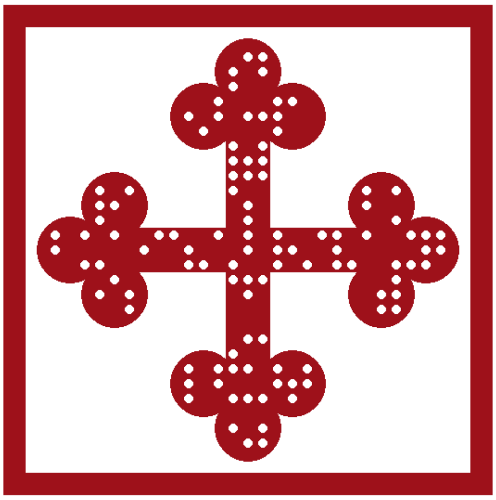


Figure 5.5: Lycacode example: final code

The external square is used to detect the code and to correct perspective. Its position and dimensions, which have been chosen quite arbitrarily, are visualized on figure 5.6. In fact, only the position and size of the inner border is relevant in the decoding process.

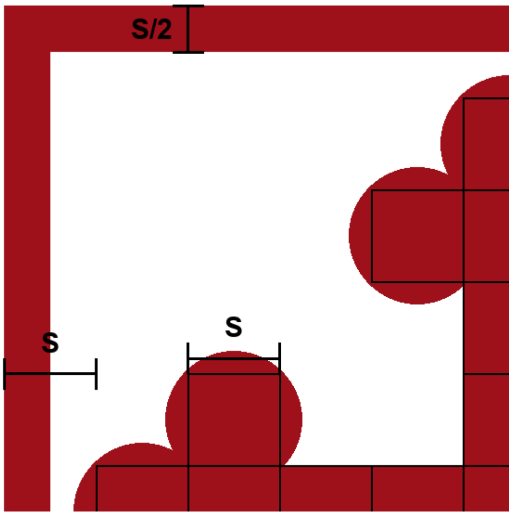


Figure 5.6: Lycacode frame dimensions

Chapter 6

Conclusion

We have seen how engineers such as Woodland and Silver have built the basis for a barcode system that optimises many processes, and how these barcodes provide an easy, fast and reliable way of encoding data. These values, ease of use, speed and reliability, are most certainly what the development of new technologies is all about. The sheer number of barcodes currently in use around the world is proof of their ingenuity. Following the same success, QR-Codes conquered the world and became part of our daily lives as we encounter them everywhere.

Because they have become standard elements in our society, they are often disregarded and wrapped in mystery, like magical tags instantly recognizable by our devices. This work is obviously not an exhaustive list of all codes that exist. Many other types can be commonly found, such as PDF-417 or Aztec codes, both of which also use the Reed-Solomon algorithm for error-correction, or Postnet, a specialized type of barcodes used by the United States Postal Service.

The Lycacode described in chapter 5 was primarily designed for education purposes, as a way to put in practice the formerly explained principles, but could well have a real application in the Collège.

Chapter 7

Personal review

My interest for computers and programming has led me to choose this topic for my work and it is not a choice I regret.

QR-Codes were a subject I personally wanted to understand for some time but never got round to it. Having the possibility to use my knowledge in Python to apply what I had researched was also very satisfying and self-rewarding. I hope I succeeded in trying to explain the inner workings and principles making these great inventions possible. I appreciated the great freedom both on the subject's choice and the realization. Apart from learning about barcodes and QR-Codes, I have also had the opportunity to put to use my English language skills. Using LaTeX helped practising problem solving as many things can be done in multiple ways and I needed to find the best or easiest method, especially regarding tables.

The Python scripts also represent a great part of this work. While not perfect implementations, they offer a better understanding of the sometimes abstract concepts discussed and allow the generation steps to be visualized personally, using any value.

Overall, I am pleased with the fruit of my research and understanding. My only regret would be that I have not yet been able to fully understand the intricacies of Reed-Solomon codes, but I hope that one day I will.

Finally, I would like to thank Mr. Erspamer for his availability, his support and his insightful feedbacks. I must also credit my brother, who allowed me to obtain some documents such as the ISO standards[5][4] and who helped with the printing of my work.

Bibliography

[1] Computerphile. *Reed Solomon Encoding - Computerphile*. en. 20th Feb. 2019. URL: https://www.youtube.com/watch?v=fBRMaEAFLE0.

[2] William A. Geisel. *Tutorial on Reed-Solomon Error Correction Coding*. en. Tech. rep. NASA Lyndon B. Johnson Space Center Houston, TX, United States, 1st Aug. 1990. URL: https://ntrs.nasa.gov/api/citations/19900019023/downloads/19900019023.pdf.

[3] *History of QR Code*. en. URL: https://www.qrcode.com/en/history/ (visited on 06/04/2022).

[4] *Information technology — Automatic identification and data capture techniques — Bar code symbology — QR Code*. Standard. International Organization for Standardization, 2015. URL: https://www.iso.org/standard/62021.html.

[5] *Information technology — Automatic identification and data capture techniques — Code 39 bar code symbology specification*. Standard. International Organization for Standardization, 2007. URL: https://www.iso.org/standard/43897.html.

[6] George Markowsky. *information theory*. Ed. by Encyclopedia Britannica. URL: https://www.britannica.com/science/information-theory (visited on 04/09/2022).

[7] I. S. Reed and G. Solomon. 'Polynomial Codes Over Certain Finite Fields'. In: *Journal of the Society for Industrial and Applied Mathematics* 8.2 (1960), pp. 300–304. DOI: 10.1137/0108018. eprint: https://doi.org/10.1137/0108018. URL: https://doi.org/10.1137/0108018.

[8] Wikipedia contributors. *Chien search — Wikipedia, The Free Encyclopedia*. 2020. URL: https://en.wikipedia.org/w/index.php?title=Chien_search&oldid=990999150 (visited on 03/09/2022).

[9] Wikipedia contributors. *Hamming code — Wikipedia, The Free Encyclopedia*. 2022. URL: https://en.wikipedia.org/w/index.php?title=Hamming_code&oldid=1065900025 (visited on 15/08/2022).

[10] Wikipedia contributors. *KarTrak — Wikipedia, The Free Encyclopedia*. 2021. URL: https://en.wikipedia.org/w/index.php?title=KarTrak&oldid=1037680217 (visited on 15/08/2022).

[11] Wikipedia Contributors. *Reed–Solomon error correction — Wikipedia, The Free Encyclopedia*. 2022. URL: https://en.wikipedia.org/w/index.php?title=Reed%E2%80%93Solomon_error_correction&oldid=1100051467 (visited on 03/08/2022).

[12] Wikiversity. *Reed–Solomon codes for coders — Wikiversity*, 2022. URL: https://en.wikiversity.org/w/index.php?title=Reed-Solomon_codes_for_coders&oldid=2387659 (visited on 06/04/2022).

[13] Norman Woodland and Bernard Silver. EN. U.S. pat. 2612994. Oct. 1952. URL: https://worldwide.espacenet.com/patent/search/family/022402610/publication/US2612994A?q=pn%3DUS2612994.

Appendix A

Python code base display module

All figures, LaTeX and Python files are available on the dedicated GitHub repository[1]

This module is used by the other generator scripts to display codes.

base.py

```
#!/usr/bin/env python
# -*- coding: utf-8 -*-
"""
This module provides a base class to display codes and enable saving

(C) 2022 Louis Heredero  louis.heredero@edu.vs.ch
"""

import pygame

class Base:
    def __init__(self, width, height, caption):
        pygame.init()

        pygame.display.set_caption(caption)
        self.w = pygame.display.set_mode([width, height])

        self.controls([
            "CTRL + S: save as",
            "ESC: quit"
        ])
```

[1] https://github.com/LordBaryhobal/5D_Heredero_Louis_TM2022

```python
def controls(self, controls, margin=2):
    longest = max(list(map(len, controls))+[10])
    print("┌─" + "─"*(longest+margin) + "─┐")

    _ = "\x1b[1;4mControls:\x1b[0m"
    _ += " "*(longest+margin-9)
    print(f"│ " + _ + " │")
    for c in controls:
        print("│ " + " "*margin + c.ljust(longest) + " │")
    print("└─" + "─"*(longest+margin) + "─┘")

def main(self):
    pygame.display.flip()

    stop = False
    while not stop:
        event = pygame.event.wait()
        # ESC or close button -> quit
        if event.type == pygame.QUIT:
            stop = True

        elif event.type == pygame.KEYDOWN:
            if event.key == pygame.K_ESCAPE:
                stop = True

            # CTRL+S -> save image
            elif event.key == pygame.K_s and \
                 event.mod & pygame.KMOD_CTRL:
                self.save()

def save(self):
    path = input("Save as: ")
    pygame.image.save(self.w, path)
```

Appendix B

Code 39 python implementation

code39.py

```
#!/usr/bin/env python
# -*- coding: utf-8 -*-
"""
This module can generate Code-39 barcodes

(C) 2022 Louis Heredero  louis.heredero@edu.vs.ch
"""

import pygame

code39_dict = {
    "A": "100001001", "B": "001001001",
    "C": "101001000", "D": "000011001",
    "E": "100011000", "F": "001011000",
    "G": "000001101", "H": "100001100",
    "I": "001001100", "J": "000011100",
    "K": "100000011", "L": "001000011",
    "M": "101000010", "N": "000010011",
    "O": "100010010", "P": "001010010",
    "Q": "000000111", "R": "100000110",
    "S": "001000110", "T": "000010110",
    "U": "110000001", "V": "011000001",
    "W": "111000000", "X": "010010001",
    "Y": "110010000", "Z": "011010000",
    "0": "000110100", "1": "100100001",
    "2": "001100001", "3": "101100000",
    "4": "000110001", "5": "100110000",
    "6": "001110000", "7": "000100101",
```

```python
    "8": "100100100", "9": "001100100",
    " ": "011000100", "-": "010000101",
    "$": "010101000", "%": "000101010",
    ".": "110000100", "/": "010100010",
    "+": "010001010", "*": "010010100"
}

def code39(text):
    text = text.upper()
    text = list(map(lambda c: code39_dict[c], text))
    return "0".join(text)

def draw_barcode(barcode, win):
    barcode = list(map(int, barcode))
    width = win.get_width()*0.8
    height = win.get_height()*0.5
    thicks = sum(barcode)
    thins = len(barcode)-thicks
    bar_w = width/(thicks*2+thins)

    win.fill((255,255,255))
    x = win.get_width()*0.1
    y = win.get_height()*0.25

    for i, c in enumerate(barcode):
        w = 2*bar_w if c else bar_w
        if i%2 == 0:
            pygame.draw.rect(win, (0,0,0), [x, y, w, height])

        x += w

if __name__ == "__main__":
    import base

    b = base.Base(800, 500, "Code-39 barcode generator")

    barcode = code39("*CODE-39*")
    draw_barcode(barcode, b.w)

    b.main()
```

Appendix C

EAN python implementation

ean.py

```
#!/usr/bin/env python
# -*- coding: utf-8 -*-
"""
This module can generate EAN-8 and EAN-13 barcodes

(C) 2022 Louis Heredero  louis.heredero@edu.vs.ch
"""

import pygame

A = [
    0b0001101,
    0b0011001,
    0b0010011,
    0b0111101,
    0b0100011,
    0b0110001,
    0b0101111,
    0b0111011,
    0b0110111,
    0b0001011
]

# XOR 0b1111111
C = list(map(lambda a: a^127, A))

# Reverse bit order
B = list(map(lambda c: int(f"{c:07b}"[::-1],2), C))
```

```python
ean13_patterns = [
    "AAAAAA",
    "AABABB",
    "AABBAB",
    "AABBBA",
    "ABAABB",
    "ABBAAB",
    "ABBBAA",
    "ABABAB",
    "ABABBA",
    "ABBABA"
]

def bin_list(n):
    return list(map(int,f"{n:07b}"))

def luhn(digits):
    checksum = sum([
        digits[-i-1]*(3-i%2*2)
        for i in range(len(digits))
    ])
    ctrl_key = 10 - checksum%10
    if ctrl_key == 10:
        ctrl_key = 0

    return ctrl_key

def ean8(digits):
    digits.append(luhn(digits))
    elmts = []

    elmts += [1,0,1] #delimiter
    for digit in digits[:4]:
        elmts += bin_list(A[digit])

    elmts += [0,1,0,1,0] #middle delimiter
    for digit in digits[4:]:
        elmts += bin_list(C[digit])

    elmts += [1,0,1] #delimiter
    return elmts

def ean13(digits):
    pattern = ean13_patterns[digits[0]]
    digits.append(luhn(digits))
    elmts = []
```

```python
    elmts += [1,0,1] #delimiter
    for d in range(1,7):
        _ = A if pattern[d-1] == "A" else B
        digit = digits[d]
        elmts += bin_list(_[digit])

    elmts += [0,1,0,1,0] #middle delimiter
    for digit in digits[7:]:
        elmts += bin_list(C[digit])

    elmts += [1,0,1] #delimiter
    return elmts

def draw_barcode(barcode, win):
    width = win.get_width()*0.8
    height = win.get_height()*0.5
    bar_w = width/len(barcode)
    rnd_bar_w = round(bar_w)

    win.fill((255,255,255))
    x = win.get_width()*0.1
    y = win.get_height()*0.25

    for c in barcode:
        if c:
            pygame.draw.rect(win, (0,0,0), [x, y, rnd_bar_w, height])

        x += bar_w

if __name__ == "__main__":
    import base

    b = base.Base(800, 500, "EAN-8 / EAN-13 barcode generator")

    #barcode = ean8([8,4,2,7,3,7,2])
    barcode = ean13([9,7,8,2,9,4,0,6,2,1,0,5])

    draw_barcode(barcode, b.w)

    b.main()
```

Appendix D

QR-Code tables

Index	Char	Index	Char	Index	Char
0	0	15	F	30	U
1	1	16	G	31	V
2	2	17	H	32	W
3	3	18	I	33	X
4	4	19	J	34	Y
5	5	20	K	35	Z
6	6	21	L	36	*space*
7	7	22	M	37	$
8	8	23	N	38	%
9	9	24	O	39	*
10	A	25	P	40	+
11	B	26	Q	41	-
12	C	27	R	42	.
13	D	28	S	43	/
14	E	29	T	44	:

Table D.1: List of alphanumerical characters

Table D.2: Version capacities

Version	Correction level	Numerical	Alphanumerical	Byte	Kanji
1	L	41	25	17	10
	M	34	20	14	8
	Q	27	16	11	7
	H	17	10	7	4
2	L	77	47	32	20
	M	63	38	26	16
	Q	48	29	20	12
	H	34	20	14	8
3	L	127	77	53	32
	M	101	61	42	26
	Q	77	47	32	20
	H	58	35	24	15
4	L	187	114	78	48
	M	149	90	62	38
	Q	111	67	46	28
	H	82	50	34	21
5	L	255	154	106	65
	M	202	122	84	52
	Q	144	87	60	37
	H	106	64	44	27
6	L	322	195	134	82
	M	255	154	106	65
	Q	178	108	74	45
	H	139	84	58	36
7	L	370	224	154	95
	M	293	178	122	75
	Q	207	125	86	53
	H	154	93	64	39
8	L	461	279	192	118
	M	365	221	152	93
	Q	259	157	108	66
	H	202	122	84	52
9	L	552	335	230	141
	M	432	262	180	111
	Q	312	189	130	80
	H	235	143	98	60

Continued on next page

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10	L	652	395	271	167
	M	513	311	213	131
	Q	364	221	151	93
	H	288	174	119	74
11	L	772	468	321	198
	M	604	366	251	155
	Q	427	259	177	109
	H	331	200	137	85
12	L	883	535	367	226
	M	691	419	287	177
	Q	489	296	203	125
	H	374	227	155	96
13	L	1022	619	425	262
	M	796	483	331	204
	Q	580	352	241	149
	H	427	259	177	109
14	L	1101	667	458	282
	M	871	528	362	223
	Q	621	376	258	159
	H	468	283	194	120
15	L	1250	758	520	320
	M	991	600	412	254
	Q	703	426	292	180
	H	530	321	220	136
16	L	1408	854	586	361
	M	1082	656	450	277
	Q	775	470	322	198
	H	602	365	250	154
17	L	1548	938	644	397
	M	1212	734	504	310
	Q	876	531	364	224
	H	674	408	280	173
18	L	1725	1046	718	442
	M	1346	816	560	345
	Q	948	574	394	243
	H	746	452	310	191

Continued on next page

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19	L	1903	1153	792	488
	M	1500	909	624	384
	Q	1063	644	442	272
	H	813	493	338	208
20	L	2061	1249	858	528
	M	1600	970	666	410
	Q	1159	702	482	297
	H	919	557	382	235
21	L	2232	1352	929	572
	M	1708	1035	711	438
	Q	1224	742	509	314
	H	969	587	403	248
22	L	2409	1460	1003	618
	M	1872	1134	779	480
	Q	1358	823	565	348
	H	1056	640	439	270
23	L	2620	1588	1091	672
	M	2059	1248	857	528
	Q	1468	890	611	376
	H	1108	672	461	284
24	L	2812	1704	1171	721
	M	2188	1326	911	561
	Q	1588	963	661	407
	H	1228	744	511	315
25	L	3057	1853	1273	784
	M	2395	1451	997	614
	Q	1718	1041	715	440
	H	1286	779	535	330
26	L	3283	1990	1367	842
	M	2544	1542	1059	652
	Q	1804	1094	751	462
	H	1425	864	593	365
27	L	3517	2132	1465	902
	M	2701	1637	1125	692
	Q	1933	1172	805	496
	H	1501	910	625	385

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28	L	3669	2223	1528	940
	M	2857	1732	1190	732
	Q	2085	1263	868	534
	H	1581	958	658	405
29	L	3909	2369	1628	1002
	M	3035	1839	1264	778
	Q	2181	1322	908	559
	H	1677	1016	698	430
30	L	4158	2520	1732	1066
	M	3289	1994	1370	843
	Q	2358	1429	982	604
	H	1782	1080	742	457
31	L	4417	2677	1840	1132
	M	3486	2113	1452	894
	Q	2473	1499	1030	634
	H	1897	1150	790	486
32	L	4686	2840	1952	1201
	M	3693	2238	1538	947
	Q	2670	1618	1112	684
	H	2022	1226	842	518
33	L	4965	3009	2068	1273
	M	3909	2369	1628	1002
	Q	2805	1700	1168	719
	H	2157	1307	898	553
34	L	5253	3183	2188	1347
	M	4134	2506	1722	1060
	Q	2949	1787	1228	756
	H	2301	1394	958	590
35	L	5529	3351	2303	1417
	M	4343	2632	1809	1113
	Q	3081	1867	1283	790
	H	2361	1431	983	605
36	L	5836	3537	2431	1496
	M	4588	2780	1911	1176
	Q	3244	1966	1351	832
	H	2524	1530	1051	647

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37	L	6153	3729	2563	1577
	M	4775	2894	1989	1224
	Q	3417	2071	1423	876
	H	2625	1591	1093	673
38	L	6479	3927	2699	1661
	M	5039	3054	2099	1292
	Q	3599	2181	1499	923
	H	2735	1658	1139	701
39	L	6743	4087	2809	1729
	M	5313	3220	2213	1362
	Q	3791	2298	1579	972
	H	2927	1774	1219	750
40	L	7089	4296	2953	1817
	M	5596	3391	2331	1435
	Q	3993	2420	1663	1024
	H	3057	1852	1273	784

Table D.3: Error correction characteristics

Version	Correction level	Data codewords	Error correction codewords per block	Blocks in group 1	Data codewords per group 1 blocks	Blocks in group 2	Data codewords per group 2 blocks
1	L	19	7	1	19	0	0
	M	16	10	1	16	0	0
	Q	13	13	1	13	0	0
	H	9	17	1	9	0	0
2	L	34	10	1	34	0	0
	M	28	16	1	28	0	0
	Q	22	22	1	22	0	0
	H	16	28	1	16	0	0
Ver	Level	Data CW	EC CW /B	Blocks G1	CW G1	Blocks G2	CW G2

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Ver	Level	Data CW	EC CW /B	Blocks G1	CW G1	Blocks G2	CW G2
3	L	55	15	1	55	0	0
	M	44	26	1	44	0	0
	Q	34	18	2	17	0	0
	H	26	22	2	13	0	0
4	L	80	20	1	80	0	0
	M	64	18	2	32	0	0
	Q	48	26	2	24	0	0
	H	36	16	4	9	0	0
5	L	108	26	1	108	0	0
	M	86	24	2	43	0	0
	Q	62	18	2	15	2	16
	H	46	22	2	11	2	12
6	L	136	18	2	68	0	0
	M	108	16	4	27	0	0
	Q	76	24	4	19	0	0
	H	60	28	4	15	0	0
7	L	156	20	2	78	0	0
	M	124	18	4	31	0	0
	Q	88	18	2	14	4	15
	H	66	26	4	13	1	14
8	L	194	24	2	97	0	0
	M	154	22	2	38	2	39
	Q	110	22	4	18	2	19
	H	86	26	4	14	2	15
9	L	232	30	2	116	0	0
	M	182	22	3	36	2	37
	Q	132	20	4	16	4	17
	H	100	24	4	12	4	13
10	L	274	18	2	68	2	69
	M	216	26	4	43	1	44
	Q	154	24	6	19	2	20
	H	122	28	6	15	2	16
11	L	324	20	4	81	0	0
	M	254	30	1	50	4	51
	Q	180	28	4	22	4	23
	H	140	24	3	12	8	13
Ver	Level	Data CW	EC CW /B	Blocks G1	CW G1	Blocks G2	CW G2

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Ver	Level	Data CW	EC CW /B	Blocks G1	CW G1	Blocks G2	CW G2
12	L	370	24	2	92	2	93
	M	290	22	6	36	2	37
	Q	206	26	4	20	6	21
	H	158	28	7	14	4	15
13	L	428	26	4	107	0	0
	M	334	22	8	37	1	38
	Q	244	24	8	20	4	21
	H	180	22	12	11	4	12
14	L	461	30	3	115	1	116
	M	365	24	4	40	5	41
	Q	261	20	11	16	5	17
	H	197	24	11	12	5	13
15	L	523	22	5	87	1	88
	M	415	24	5	41	5	42
	Q	295	30	5	24	7	25
	H	223	24	11	12	7	13
16	L	589	24	5	98	1	99
	M	453	28	7	45	3	46
	Q	325	24	15	19	2	20
	H	253	30	3	15	13	16
17	L	647	28	1	107	5	108
	M	507	28	10	46	1	47
	Q	367	28	1	22	15	23
	H	283	28	2	14	17	15
18	L	721	30	5	120	1	121
	M	563	26	9	43	4	44
	Q	397	28	17	22	1	23
	H	313	28	2	14	19	15
19	L	795	28	3	113	4	114
	M	627	26	3	44	11	45
	Q	445	26	17	21	4	22
	H	341	26	9	13	16	14
20	L	861	28	3	107	5	108
	M	669	26	3	41	13	42
	Q	485	30	15	24	5	25
	H	385	28	15	15	10	16
Ver	Level	Data CW	EC CW /B	Blocks G1	CW G1	Blocks G2	CW G2

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Ver	Level	Data CW	EC CW /B	Blocks G1	CW G1	Blocks G2	CW G2
21	L	932	28	4	116	4	117
	M	714	26	17	42	0	0
	Q	512	28	17	22	6	23
	H	406	30	19	16	6	17
22	L	1006	28	2	111	7	112
	M	782	28	17	46	0	0
	Q	568	30	7	24	16	25
	H	442	24	34	13	0	0
23	L	1094	30	4	121	5	122
	M	860	28	4	47	14	48
	Q	614	30	11	24	14	25
	H	464	30	16	15	14	16
24	L	1174	30	6	117	4	118
	M	914	28	6	45	14	46
	Q	664	30	11	24	16	25
	H	514	30	30	16	2	17
25	L	1276	26	8	106	4	107
	M	1000	28	8	47	13	48
	Q	718	30	7	24	22	25
	H	538	30	22	15	13	16
26	L	1370	28	10	114	2	115
	M	1062	28	19	46	4	47
	Q	754	28	28	22	6	23
	H	596	30	33	16	4	17
27	L	1468	30	8	122	4	123
	M	1128	28	22	45	3	46
	Q	808	30	8	23	26	24
	H	628	30	12	15	28	16
28	L	1531	30	3	117	10	118
	M	1193	28	3	45	23	46
	Q	871	30	4	24	31	25
	H	661	30	11	15	31	16
29	L	1631	30	7	116	7	117
	M	1267	28	21	45	7	46
	Q	911	30	1	23	37	24
	H	701	30	19	15	26	16
Ver	Level	Data CW	EC CW /B	Blocks G1	CW G1	Blocks G2	CW G2

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Ver	Level	Data CW	EC CW /B	Blocks G1	CW G1	Blocks G2	CW G2
30	L	1735	30	5	115	10	116
	M	1373	28	19	47	10	48
	Q	985	30	15	24	25	25
	H	745	30	23	15	25	16
31	L	1843	30	13	115	3	116
	M	1455	28	2	46	29	47
	Q	1033	30	42	24	1	25
	H	793	30	23	15	28	16
32	L	1955	30	17	115	0	0
	M	1541	28	10	46	23	47
	Q	1115	30	10	24	35	25
	H	845	30	19	15	35	16
33	L	2071	30	17	115	1	116
	M	1631	28	14	46	21	47
	Q	1171	30	29	24	19	25
	H	901	30	11	15	46	16
34	L	2191	30	13	115	6	116
	M	1725	28	14	46	23	47
	Q	1231	30	44	24	7	25
	H	961	30	59	16	1	17
35	L	2306	30	12	121	7	122
	M	1812	28	12	47	26	48
	Q	1286	30	39	24	14	25
	H	986	30	22	15	41	16
36	L	2434	30	6	121	14	122
	M	1914	28	6	47	34	48
	Q	1354	30	46	24	10	25
	H	1054	30	2	15	64	16
37	L	2566	30	17	122	4	123
	M	1992	28	29	46	14	47
	Q	1426	30	49	24	10	25
	H	1096	30	24	15	46	16
38	L	2702	30	4	122	18	123
	M	2102	28	13	46	32	47
	Q	1502	30	48	24	14	25
	H	1142	30	42	15	32	16
Ver	Level	Data CW	EC CW /B	Blocks G1	CW G1	Blocks G2	CW G2

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Ver	Level	Data CW	EC CW /B	Blocks G1	CW G1	Blocks G2	CW G2
39	L	2812	30	20	117	4	118
	M	2216	28	40	47	7	48
	Q	1582	30	43	24	22	25
	H	1222	30	10	15	67	16
40	L	2956	30	19	118	6	119
	M	2334	28	18	47	31	48
	Q	1666	30	34	24	34	25
	H	1276	30	20	15	61	16

Table D.4: Alignment pattern locations

Version	Central x and y coordinates						
1							
2	6	18					
3	6	22					
4	6	26					
5	6	30					
6	6	34					
7	6	22	38				
8	6	24	42				
9	6	26	46				
10	6	28	50				
11	6	30	54				
12	6	32	58				
13	6	34	62				
14	6	26	46	66			
15	6	26	48	70			
16	6	26	50	74			
17	6	30	54	78			
18	6	30	56	82			
19	6	30	58	86			
20	6	34	62	90			
21	6	28	50	72	94		
22	6	26	50	74	98		
Version	Central x and y coordinates						

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Version	Central x and y coordinates						
23	6	30	54	78	102		
24	6	28	54	80	106		
25	6	32	58	84	110		
26	6	30	58	86	114		
27	6	34	62	90	118		
28	6	26	50	74	98	122	
29	6	30	54	78	102	126	
30	6	26	52	78	104	130	
31	6	30	56	82	108	134	
32	6	34	60	86	112	138	
33	6	30	58	86	114	142	
34	6	34	62	90	118	146	
35	6	30	54	78	102	126	150
36	6	24	50	76	102	128	154
37	6	28	54	80	106	132	158
38	6	32	58	84	110	136	162
39	6	26	54	82	110	138	166
40	6	30	58	86	114	142	170